B.Sc. Part-III (Semester-VI) Examination

MATHEMATICS

(Linear Algebra)

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt this question once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) S is a non-empty subset of vector space V, then the smallest subspace of V containing S is :

(a) S (b) {S}

(c) [S] (d) None

(ii) Let U and V be finite dimensional vector spaces and T : U → V be a linear map one-one and onto, then :

(a) dim U = dim V (b) dim U ≠ dim V

(c) U = V (d) U ≠ d

(iii) Let W is subspace of vector space V. Then $\{f \in \hat{V}/f(w) = 0, \forall w \in W\}$ is called as :

(a) Hilatory of W (b) Annihilator of W

(c) Dual space of W (d) None

(iv) The normalized vector (1, –2, 5) is :

(a) (1, –2, 5) (b) $\left(\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{.5}{\sqrt{30}}\right)$

(c) $\left(\frac{1}{2}, -1, \frac{5}{2}\right)$ (d) $\left(\frac{1}{5}, \frac{-2}{5}, 1\right)$

(v) In IPS V(F) the relation $\| u + v \|^2 + \| u - v \|^2 = 2 (\| u \|^2 + \| v \|^2)$ is called as :

(a) Schwartz inequality (b) Triangle law

(c) Parallelogram law (d) Bessels inequality

(vi) For two subspaces U and W of V(F), V = U ⊕ W ⇔ ....

(a) U ∩ W = {0} (b) V = U + W

(c) U ∩ W = {0} and V = U + W (d) None of these

(vii) Let $T : M \to N$ be an R-homomorphism. If B is a submodule of N, then :

(a) $T^{-1}(B)$ is submodule of N
(b) $T^{-1}(B)$ is submodule of M
(c) $T^{-1}(B)$ is kernel of R-homomorphism
(d) $T^{-1}(B) = T(M)$

(viii) If $T : U \to V$ then the set $\{T(u) \mid u \in U\}$ = ...

(a) Ker T
(b) R(u)
(c) R(T)
(d) None of these

(ix) If $\| V \| = 1$, then V is called :

(a) Normalised
(b) Orthonormal
(c) Scalar inner product
(d) Standard inner product

(x) If $\hat{V}$ is n-dimensional, then the dimension of V is :

(a) Less than n
(b) Greater than n
(c) Equal n
(d) Zero 10

UNIT—I

2. (a) Let U and W be two subspaces of a vector space V and $Z = U + W$. Then prove that $Z = U \oplus W$ iff $z \in Z$, $z = u + w$ is unique representation for $u \in U$ and $w \in W$. 5

(b) Extend the linearly independent set $\{(1, 1, 1, 1)\ (1, 2, 1, 2)\}$ in V_4 to a basis for V_4. 5

3. (p) If U and W are finite dimensional subspaces of vector space V, then prove that :

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W).$$ 5

(q) Let R^+ be the set of all positive real numbers. Define the operations of addition $\oplus$ and scalar multiplication $\otimes$ as follows :

$$u \oplus v = u \cdot v \ \forall\, u, v \in R^+$$

and $\alpha \otimes u = u^{\alpha}, \forall\, u \in R^+$ and $\alpha \in R$

Prove that R^+ is a real vector space. 5

UNIT—II

4. (a) If U, V is a vector space over a field F and $T : U \to V$ be a linear, then prove that :

$$T(\alpha_1 u_1 + \alpha_2 u_2 + \ldots\ldots + \alpha_n u_n) = \alpha_1 T(u_1) + \alpha_2 T(u_2) + \ldots\ldots + \alpha_n T(u_n)$$

$$\forall\, u_i \in U, \alpha_i \in F, 1 \le i \le n \text{ and } n \in N.$$ 2

(b) Let $T : V_4 \to V_3$ be a linear map defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$.

Verify Rank-nullity theorem. 4

(c) Find the matrix of the linear map $T : V_2 \to V_3$ defined by $T(x, y) = (-x + 2y, y, -3x + 3y)$ related to the bases $B_1 = \{(1, 2), (-2, 1)\}$ and $B_2 = \{(-1, 0, 2), (1, 2, 3), (1, -1, 1)\}$. 4

5. (p) Let U and V be vector spaces over the same field F. Then prove that function $T : U \to V$ is linear iff $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v), \forall \alpha, \beta \in F$ and $u, v \in U$. 5

(q) If matrix of a linear map T with respect to bases B_1 and B_2 is :

$$\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

where $B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and $B_2 = \{(1, 0), (2, -1)\}$, then find $T(x, y, z)$. 5

UNIT—III

6. (a) Let V be the space of all real valued continuous functions of real variable. Define $T : V \to V$ by

$$(Tf)(x) = \int_0^x f(t)\, dt, \forall f \in V, x \in R.$$

Show that T has no eigen value. 5

(b) Prove that if V be a finite dimensional vector space over F and $v(\neq 0) \in V$, then $\exists f \in \hat{V}$ such that $f(v) \neq 0$. 5

7. (p) If W_1 and W_2 are subspaces of a finite dimensional vector space V, show that $A(W_1 + W_2) = A(W_1) \cap A(W_2)$. 5

(q) If K_λ is eigenspace, then prove that K_λ is a subspace of vector space V. 3

(r) Define characteristic root and characteristic vector. 2

UNIT—IV

8. (a) In $F^{(n)}$ define for $u = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n)$ and $v = (\beta_1, \beta_2 ...... \beta_n)$

$$(u, v) = \alpha_1\overline{\beta}_1 + \alpha_2\overline{\beta}_2 + ... + \alpha_n\overline{\beta}_n.$$

Show that this defines an inner product. 4

(b) If $\{x_1, x_2, x_3, ... x_n\}$ be an orthogonal set, then prove that :

$$\| x_1 + x_2 + x_3 + ... + x_n \|^2 = \| x_1 \|^2 + \| x_2 \|^2 + ... + \| x_n \|^2$$ 4

(c) Prove that orthogonal complement i.e. $W^\perp$ is subspace of V. 2

9. (p) If $\{w_1, w_2, ... w_m\}$ is an orthonormal set in V, then $\sum_{i=1}^{m} |(w_i, v)|^2 \leq \| v \|^2$ for any $v \in V$. 4

(q) If V is a finite dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp} = W$. 4

(r) (i) Define inner product in vector space. 1

(ii) Define orthogonal set. 1

UNIT—V

10. (a) Let A be a submodule of an R-module M and T is a mapping from M into M/A defined by $T_m = A + m, \forall m \in M$. Then prove that T is an R-homomorphism of M into M/A and ker T = A. 5

(b) Let T be a homomorphism of an R-module M to an R-module H. Prove that T is one-one iff ker T = $\{0\}$. 3

(c) Define :

(i) Submodule

(ii) Unital R-module. 2

11. (p) If A and B are submodules of M, then prove that $\frac{A+B}{B}$ is isomorphic to $\frac{A}{A \cap B}$. 6

(q) Prove that arbitrary intersection of submodules of a module is a submodule. 4

B.Sc. Part-III (Semester-VI) Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The interval $ds^2 = -(dx')^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$ is said to be space like if : 1

(a) $ds^2 > 0$ (b) $ds^2 < 0$

(c) $ds^2 = 0$ (d) None of these

(ii) The electric and magnetic field strengths E and H are invariant under : 1

(a) Galilean Transformations (b) Laplace Transformations

(c) Fourier Transformations (d) Guage Transformations

(iii) $A^i = (\bar{A}, \phi) = (Ax, Ay, Az, \phi)$ is a four potential then : 1

(a) $A_i = (\bar{A}, \phi)$ (b) $A_i = (\bar{A}, -\phi)$

(c) $A_i = (-\bar{A}, \phi)$ (d) $A_i = (-\bar{A}, -\phi)$

(iv) $A^r = (A^1, A^2, A^3, A^4)$ is a four vector or four dimensional vector where $A^2 < 0$ then A^r is : 1

(a) Time like (b) Null or light like

(c) Space like (d) None of these

(v) Covariant tensor of rank one T'_r is defined as : 1

(a) $T'_r = \frac{\partial x'^r}{\partial x^5} T_s$ (b) $T'_r = \frac{\partial x'^r}{\partial x^5} T_r$

(c) $T'_r = \frac{\partial x^5}{\partial x'^r} T_s$ (d) $T'_r = \frac{\partial x^5}{\partial x'^r} T_r$

(vi) The special Lorentz transformations will reduce to simple Galilean transformations when : 1

(a) $V = C$ (b) $C << V$

(c) $V << C$ (d) None of these

(vii) The electromagnetic field tensor (or Maxwell tensor) F_{ij} is defined as : 1

(a) $F_{ij} = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ (b) $F_{ij} = \frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}$

(c) $F_{ij} = \frac{\partial A_i}{\partial x^j} + \frac{\partial A_j}{\partial x^i}$ (d) None of these

(viii) The transformations $\bar{r}' = \bar{r} - \bar{v}t$ and $t' = t$ are : 1

(a) Laplace transformations (b) Lorentz transformations

(c) Galilean transformations (d) None of these

(ix) If $\overline{A}$ is a vector potential then the magnetic field is : 1

(a) $\overline{H} = \text{div}.\overline{A}$ (b) $\overline{H} = \text{Curl}\,\overline{A}$

(c) $\overline{H} = \text{div}.(\text{Curl}\,\overline{A})$ (d) None of these

(x) Four velocity of a particle is : 1

(a) a unit space-like vector (b) a unit time-like vector

(c) a unit light-like vector (d) None of these

UNIT—I

2. (a) Obtain Galilean transformation equations for two inertial frames in relative motion. 3

(b) Show that simultaneity is relative in special relativity. 3

(c) Show that the electromagnetic wave equation :

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0$$

is not invariant under the Galilean transformations. 4

3. (p) Discuss the geometrical interpretation of Lorentz transformations. 4

(q) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(r) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 2

UNIT—II

4. (a) Obtain the transformations for the velocity of a particle under special Lorentz transformations. 5

(b) If $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems s and s' respectively where s' is moving with velocity v relative to s along the **XX'** axis then show that :

$$\tan\theta' = \frac{\sin\theta\left(1-\frac{v^2}{c^2}\right)^{1/2}}{\cos\theta - \frac{v}{u}}$$

and

$$u'^2 = \frac{u^2\left[1 - 2\frac{v}{u}\cos\theta + \frac{v^2}{u^2} - \frac{v^2}{c^2}\sin^2\theta\right]}{\left(1 - \frac{uv}{c^2}\cos\theta\right)^2}$$

where θ and θ' are the angles made by u and u' with the X-axis respectively. 5

5. (p) If $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems s and s' respectively then prove that :

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}{\left(\frac{1+u'_x v}{c}\right)},$$

where s' is moving with velocity v relative to s along XX' axis. 5

(q) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

UNIT—III

6. (a) Define time-like, space-like and light-like intervals for the space time geometry of special relativity. 3

(b) Define a four tensor of the second order. Prove that :

(i) $T'^{11} = \alpha^2\left\{T^{11} - \frac{v}{c}T^{14} - \frac{v}{c}T^{41} + \frac{v^2}{c^2}T^{44}\right\}$ and

(ii) $T'^{14} = \alpha^2\left\{-\frac{v}{c}T^{11} + T^{14} + \frac{v^2}{c^2}T^{41} - \frac{v}{c}T^{44}\right\}$ 1+3+3

7. (p) Define a four vector A^r. Show that :

$A^1 = -A_1,\ A^2 = -A_2,\ A^3 = -A_3,\ A^4 = A_4.$ 1+3

(q) Prove that there exists an inertial system s' in which the two events occur at one and the same time if the interval between two events is space-like. 4

(r) Write the Lorentz transformations in index form. 2

UNIT—IV

8. (a) Deduce Einstein's mass energy equivalence relation. 5

(b) Define : Four velocity. Prove that the four velocity in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1 - u^2/c^2}}, \frac{1}{\sqrt{1 - u^2/c^2}} \right)$$

where $\bar{u} = (u_x, u_y, u_z)$ = velocity of the particle. 1+4

9. (p) Define : Four momentum vector p^i. Prove that the square of the magnitude of the four momentum vector p^i is $m^2{}_\circ c^2$. 1+4

(q) A particle is given a kinetic energy equal to n times its rest energy $m_\circ c^2$. Find speed and momentum of the particle. $\left(\text{Kinetic energy} = T = m_\circ c^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\} \right)$ 5

UNIT—V

10. (a) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m^2 c^4 + c^2 \left(p - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\phi .$$ 5

(b) Define : Current four vector. Show that $c^2 \rho^2 - J^2$ is invariant and its value is $\rho^2{}_\circ c^2$. 1+4

11. (p) Prove that the set of Maxwell's equations $\operatorname{div} . \bar{H} = 0$ and $\bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t}$ can be written as $\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0$, where F_{ij} is the electro-magnetic field tensor. 5

(q) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 1+4

B.Sc. (Part-III) Semester-VI Examination

MATHEMATICS

Linear Algebra

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt this question once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) Any superset of a linearly dependent set is : 1

(a) Linearly independent

(b) Linearly dependent

(c) Linearly independent and linearly dependent

(d) None of these

(2) If U and W are the subspaces of a vector space V(F) then $U \cup W$ is a subspace iff : 1

(a) $U \subseteq W$ or $W \subseteq U$ (b) $U \supseteq W$ or $W \supseteq U$

(c) $U \cap W = \{0\}$ (d) None of these

(3) If $T : U \to V$ be a linear map then R(T) is a subspace of : 1

(a) U (b) V

(c) $U \cap V$ (d) None of these

(4) If U, V be finite dimensional vector space and $T : U \to V$ be a linear one-one and onto map then : 1

(a) dim U = dim V (b) U = V

(c) dim U ≠ dim V (d) U ≠ V

(5) An element of dual space of V is called a : 1

(a) Linear element (b) Bilinear element

(c) Linear functional (d) None of these

(6) Eigen vectors corresponding to distinct eigen values of a square matrix are : 1

(a) Linearly independent

(b) Linearly dependent

(c) Linearly independent as well as linearly dependent

(d) None of these

(7) In an inner product space V, the inequality $|(u, v)| \leq \|u\| \cdot \|v\|$, for all $u, v \in V$ is known as : 1

(a) Triangular inequality

(b) Cauchy-Schwartz inequality

(c) Bessel's inequality

(d) None of these

(8) If W is a subspace of an inner product space V and $W^{\perp}$ is orthogonal complement of W, then : 1

(a) $W^{\perp}$ is a subspace of W

(b) $W \cap W^{\perp} = \{0\}$

(c) $W \cap W^{\perp} \neq \{0\}$

(d) None of these

(9) If A is any submodule of a R-module M, then the zero element of the quotient group M/A is : 1

(a) M

(b) A

(c) $\{0\}$

(d) None of these

(10) Let $T : M \to H$ be a homomorphism of a R-module M into R-module H, then : 1

(a) R(T) is a subset of M

(b) R(T) is a submodule of M

(c) R(T) is a submodule of H

(d) None of these

UNIT—I

2. (a) Define Linear span. If S be a non-empty subset of a vector space V, then prove that [S] is the smallest subspace of V containing S. 1+3

(b) Prove that an arbitrary intersection of subspaces of a vector space is again a subspace. 3

(c) Prove that the set of functions $\{x, |x|\}$ is L.I. in a real vector space of the continuous functions defined on $(-1, 1)$. 3

3. (p) If U and W are finite dimensional subspaces of a vector space V, then prove that :

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W).$$ 6

(q) Given two LI vectors (1, 0, 1, 0), (0, –1, 1, 0) of V_4. Find a basis of V_4 that includes these two vectors. 4

UNIT—II

4. (a) If T is a linear transformation of V_2 to V_2 defined by T(2, 1) = (3, 4), T(–3, 4) = (0, 5), then express (0, 1) as a LC of (2, 1) and (–3, 4). Hence find image of (0, 1) under T. 3

(b) Let $T : U \rightarrow V$ be a linear map. Then prove that N(T) is a subspace of U. 3

(c) Let $T : V_4 \rightarrow V_3$ be a linear map defined by :

$$T(e_1) = (1, 1, 1),\ T(e_2) = (1, -1, 1),\ T(e_3) = (1, 0, 0),\ T(e_4) = (1, 0, 1).$$

Verify Rank-Nullity theorem. 4

5. (p) If $T : U \rightarrow V$ be a non-singular linear map, then prove that $T^{-1} : V \rightarrow U$ is also a non-singular linear map. 3

(q) If the matrix of a linear map T with respect to bases B_1 and B_2 is $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

where $B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and $B_2 = \{(1, 0), (2, -1)\}$.

Find T(x, y, z). 4

(r) Find the range, kernel, rank and nullity of a matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and verify Rank-Nullity theorem. 3

UNIT—III

6. (a) Let V be the finite dimensional vector space over F. Then prove that $V \approx \hat{\hat{V}}$. 4

(b) If V is finite dimensional and $V_1 \neq V_2$ are in V, prove that there is an $f \in \hat{V}$ such that $f(V_1) \neq f(V_2)$. 3

(c) Prove that A(W) is a subspace of $\hat{V}$. 3

7. (p) Define Annihilator W. If V be a vector space over F for a subset S of V and $A(S) = \{f \in \hat{V} / f(s) = 0, \forall s \in S\}$, then prove that A(S) = A(L(S)), where L(S) is linear span of S. 1+3

(q) If U, V are finite dimensional complex vector spaces and $A : U \rightarrow V$, $B : U \rightarrow V$ are linear maps with $\alpha \in C$, then prove that $(A + B)^* = A^* + B^*$. 3

(r) If W_1 and W_2 are subspaces of a finite dimensional vector space V over F then describe $A(W_1 \cap W_2)$ in terms of $A(W_1)$ and $A(W_2)$. 3

UNIT—IV

8. (a) In an IPS V over F, prove the parallelogram law :

$$\| u + v \|^2 + \| u - v \|^2 = 2(\| u \|^2 + \| v \|^2).$$ 5

(b) Apply Gram-Schmidt method to orthonormalise the set :

$\{(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)$. 5

9. (p) Let $W^{\perp}$ be the set of orthogonal vectors in an IPS V, then prove that $W^{\perp}$ is a subspace of V. 3

(q) Let V be a finite dimensional inner product space. Then prove that V has an orthogonal set as a basis. 4

(r) In $F^{(n)}$ define, for $u = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $v = (\beta_1, \beta_2, ..., \beta_n)$,

$$(u, v) = \alpha_1\overline{\beta}_1 + \alpha_2\overline{\beta}_2 + ... + \alpha_n\overline{\beta}_n .$$

Show that this defines an inner product. 3

UNIT—V

10. (a) Let T be a homomorphic of R-module M into an R-module H. Then prove that T is one-one iff Ker T = {0}. 3

(b) If M_1 and M_2 are submodules of R-module M_1, then prove that $M_1 + M_2$ is a submodule of M. Moreover $M_1 + M_2$ is direct sum of M_1 and $M_2 \Leftrightarrow M_1 \cap M_2 = \{0\}$. 4

(c) If T is a homomorphism of an R-module M to an R-module H, then show that :

(i) T(o) = o

(ii) $T(-m) = -Tm, \forall\, m \in M$

(iii) $T(m_1 - m_2) = Tm_1 - Tm_2, \forall\, m_1, m_2 \in M$. 3

11. (p) If A and B are submodules of M, then prove that $\frac{A + B}{B}$ is isomorphic to $\frac{A}{A \cap B}$. 5

(q) Define submodule of a module. Prove that arbitrary intersection of submodules of a module is a submodule. 5

AS–1471

B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS

(Linear Algebra)

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt this question once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If S is non empty subset of vector sapce V, then L(S) is 1

(a) Largest subspace of V containing S.

(b) Smallest subspace of V containing S.

(c) Smallest subspace of V containing V.

(d) None of these.

(ii) The basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} of the vector space $R^3(R)$ is known as 1

(a) Normal basis (b) Standard basis

(c) Quotient basis (d) Hamel basis

(iii) If U, V be finite dimensional vector spaces and $T : U \to V$ be a linear, one-one and onto map, then 1

(a) dim U = dim V (b) U = V

(c) dim U ≠ dim V (d) U ≠ V

(iv) The kernel of a linear transformation $T : U \to V$ is a subset of 1

(a) U (b) V

(c) U and V (d) None of these

(v) An element of dual space of V is called a 1

(a) Linear element (b) Bilinear element

(c) Linear functional (d) None of these

(vi) Eigen vectors corresponding to distinct eigen values of a square matrix are 1

(a) Linearly independent

(b) Linearly dependent

(c) Linearly independent as well as Linearly dependent

(d) None of these.

(vii) If $\| V \| = 1$, then V is called 1

(a) Normalised (b) Orthonormal

(c) Scalar inner product (d) Standard inner product.

(viii) In an inner product space V(F), following relation : 1

$\| u + v \|^2 + \| u - v \|^2 = 2 (\| u \|^2 + \| v \|^2)$ is called ....

(a) Schwrtz's inequality (b) Triangular law

(c) Parallelogram Law (d) Bessel's inequality

(ix) If ring R has a unit element 1 and 1 . a = a, for all $a \in M$, then M is called 1

(a) Unital R-module (b) Left R-module

(c) Unique R-module (d) None of these

(x) If M is any R-module, then M and {0} are always submodules of M these are called .... submodules of M : 1

(a) Proper (b) Improper

(c) Subproper (d) Irreducible.

UNIT—I

2. (a) Let R^+ be the set of all positive real number. Define the operations of vector addition $\oplus$ and scalar multiplication $\otimes$ as follows :

$u \oplus v = uv, \forall u, v \varepsilon R^+$

and $\alpha \otimes u = u^\alpha, \forall u \varepsilon R^+ \alpha \varepsilon R$.

Prove that R^+ is a real vector space. 5

(b) Let U and W be two subspaces of a vector space V and Z = U + W. Then prove that $Z = U \oplus W \Leftrightarrow z = u + w$ is unique representation for any $z \varepsilon Z$ and for some $u \varepsilon U, \omega \varepsilon W$. 5

3. (p) Prove that the intersection of two subspaces of a vector space is again a subspace. Is this statement true for union ? 5

(q) Show that the ordered set S = {(1, 1, 0), (0, 1, 1), (1, 0, –1), (1, 1, 1)} is LD and locate one of the vectors from S that belongs to the span of the previous ones. Find also the largest LI subset of S whose span is [S]. 5

UNIT—II

4. (a) Find a linear transformation T from V_2 to V_2 s.t. T (1, 0) = (1, 1) and T (0, 1) = (–1, 2). Prove that T maps the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1) into a parrallelogram. 3

(b) Let T : U → V be a linear map. Then prove that R(T) is a subspace of V. 3

(c) Find the range, kernel, rank and nullity of the matrix :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -2 & 5 \end{bmatrix}$$

and verify Rauk-Nullity theorem. 4

5. (p) Find the matrix of the linear map T : $V_2 \rightarrow V_3$ defined by T (x, y) = (– x + 2y, y, – 3x + 3y) related to the bases

$$B_1 = \{(1, 2), (-2, 1)\}$$

and $B_2 = \{(-1, 0, 2), (1, 2, 3), (1, -1, 1)\}$. 5

(q) State and prove Rank-Nullity theorem. 5

UNIT—III

6. (a) Let V be a finite dimensional vector space over F, then prove that $V \approx \hat{V}$. 4

(b) If W_1 and W_2 are subspaces of a finite dimensional vector space V over F, then show that

$$A(W_1 \cap W_2) = A(W_1) + A(W_2).$$ 3

(c) Prove that annihitator of W = A(W) is a subspace of $\hat{V}$. 3

7. (p) Let U, V be finite dimensional complex vector spaces and A : U → V, B : U → V be linear maps of α ε C, then prove that :

(i) $(A + B)^* = A^* + B^*$,

(ii) $(\alpha A)^* = \bar{\alpha} A^*$. 3+2

(q) If W is a subspace of a finite dimensional vector space V, then prove that

$$A(A(W)) = W.$$ 5

UNIT—IV

8. (a) Let V be a set of all continuous complex valued functions on the closed interval [0, 1]. If f(t), g(t) εV, defined by

$$(f(t), g(t)) = \int_0^1 f(t) \cdot \overline{g}(t)\, dt, \text{ then}.$$

show that this defines an inner product on V. 5

(b) Using Gram-Schmidt Orthogenalisation process orthonormalise the L.I. subset {(1, 1, 1), (0, 1, 1), (0, 0, 1)} of V_3. 5

9. (p) If $\{x_1, x_2, \ldots, x_n\}$ be an orthogonal set, then prove that :

$$\| x_1 + x_2 + \ldots + x_n \|^2 = \| x_1 \|^2 + \| x_2 \|^2 + \ldots + \| x_n \|^2.$$ 3

(q) Prove that in an inner product space V,

(i) $\| \alpha u \| = | \alpha | \| u \|$,

(ii) $\| u + v \| \leq \| u \| + \| v \|$. 4

(r) If V is a finite dimensional inner product space and W is a subspace of V then show that $(W^{\perp})^{\perp} = W$. 3

UNIT—V

10. (a) Prove that arbitrary intersection of submodules of a module is a submodule. 3

(b) Let M be an R-module. Then prove the following :

(i) $\gamma \cdot 0 = 0 \quad , \forall \gamma \varepsilon R$

(ii) $-(\gamma \cdot a) = \gamma \cdot (-a) = (-\gamma) \cdot a, \ \forall \gamma \varepsilon R$ and mεM. 4

(c) If A be a submodule of unital R-module M, then prove that M/A is also unital R-module. 3

11. (p) Define R-module homomorphism. If $T : M \rightarrow H$ be an R-module homomorphism, then prove that :

(i) K(T) is a submodule of M and R(T) is submodule of H.

(ii) T is one-one $\Leftrightarrow$ K(T) = {0}. 1+4

(q) Let M be an R-module. If H and K are submodules of M with $K \subset H$. Then prove that

$$\frac{M}{H} \cong \frac{M/K}{H/K}.$$ 5

B.Sc. (Part—III) Semester-VI Examination

MATHEMATICS (NEW)

(Linear Algebra)

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative : 10

(i) A non empty subset U of a vector space V(F) is a subspace of V iff :

(a) $\alpha\beta + uv \in U$ (b) $\alpha u + \beta v \in V$

(c) $\alpha u + \beta . v \in U$ (d) $\alpha u - \beta v \in V$ for all $\alpha, \beta \in F$ and $u, v \in U$

(ii) Any subset of linearly independent set is :

(a) linearly dependent

(b) linearly dependent and linearly independent

(c) linearly independent

(d) None of these

(iii) If T : u → v is linear map then R(T) is subset of :

(a) V (b) $U \cap V$

(c) U (d) $U \cup V$

(iv) An element of dual space V is called a :

(a) Linear element (b) Linear functional

(c) Bilinear element (d) None of these

(v) If u, v be finite dimensional vector spaces and T : u → v be a linear one-one and onto map, then :

(a) dim U = dim V (b) dim U ≠ dim V

(c) U = V (d) U ≠ V

(vi) If V is the finite dimensional vector space over F then :

(a) $V \cong \hat{V}$ (b) $V \neq \hat{V}$

(c) $\hat{V} = \{0\}$ (d) None of these

(vii) If $\| V \| = 1$ then V is called :

(a) Orthogonal (b) Null vector

(c) Normalised (d) None of these

(viii) The normalised vector of (1, –2, 5) is :

(a) $\left(\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right)$ (b) $\left(\frac{1}{2}, -1, \frac{5}{2}\right)$

(c) $\left(\frac{1}{5}, \frac{-2}{5}, -1\right)$ (d) (1, –2, 5)

(ix) R-Module homomorphism is linear transformation if :

(a) R-with unit element (b) R is commutative

(c) R-is a field (d) None of these

(x) If the ring R has a unit element 1 and $1.a = a$ for all $a \in M$ then M is called :

(a) A unital R-module (b) Right R-module

(c) Left-R-module (d) None of these

UNIT—I

2. (a) Define a basis of a vector space. If $\{v_1, v_2, ..., v_n\}$ is a basis of V over F and if $w_1, w_2, ..., w_m \in V$ are L.I. over F, then prove that $m \leq n$. 1+4

(b) Define a subspace of a vector space and prove that the non empty subset U of a vector space V(F) is a subspace of V iff $\alpha u + \beta v \in U \ \forall \ \alpha, \beta \in F, u, v \in U$. 1+4

3. (p) Prove that the intersection of two subspaces of a vector space is again a subspace. Is the statement true for union ? Justify. 5

(q) Find span of S = {(1, 2, 1), (1, 1, –1), (4, 5, –2)} and then prove that (2, –1, –8) belongs to the span of S^1 but (1, –3, 5) does not belongs to span of S. 5

UNIT—II

4. (a) Let U, V are the vector spaces over a field F and $T : u \rightarrow v$ be a linear map. Then prove that :

(i) $T(0) = 0$

(ii) $T(-u) = -T(u) \ \forall \ u \in U$

(iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + ... + \alpha_n u_n) = \alpha_1 T(u_1) + \alpha_2 T(u_2) + ... + \alpha_n T(u_n)$

$\forall \ u_i \in U, \alpha_i \in F, 1 \leq i \leq n$ and $n \in N$.

5

(b) Let $T : V_4 \rightarrow V_3$ be a linear map defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Verify Rank-Nullity theorem. 5

5. (p) State and prove Rank-Nullity Theorem. 5

(q) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a matrix of linear map T with respect to bases B_1 and B_2 where $B_1 = \{(1, 1, 1), (1, 0, 0), (0, 1, 0)\}$, $B_2 = \{(1, 2, 3), (1, -1, 1), (2, 1, 1)\}$. Find $T : V_3 \rightarrow V_3$ such that $A = (T : B_1, B_2)$. 5

UNIT—III

6. (a) Let V be a finite dimensional vector space over F. Then prove that $V \approx \hat{V}$. 5

   (b) Define Annihilator of W. Prove that annihilator of W = A(W) is a subspace of $\hat{V}$. 5

7. (p) If U and V are finite dimensional complex vector spaces and $A : U \to V$, $B : U \to V$ are linear maps, then prove that (i) $(A + B)^* = A^* + B^*$, (ii) $(\alpha A)^* = \overline{\alpha} A^*$. 5

   (q) If S is a subset of a vector space V and $A(S) = \{f \in \hat{V} / f(x) = 0 \ \forall \ s \in S\}$ then prove that $A(S) = A(L(S))$ where L(S) is the linear span of S. 5

UNIT—IV

8. (a) State and prove Cauchy-Schwarz inequality. 5

   (b) (i) If $\{x_1, x_2, ..., x_n\}$ is an orthogonal set, then prove that :

   $$\| x_1 + x_2 + x_3 + ... + x_n \|^2 = \| x_1 \|^2 + \| x_2 \|^2 + ... + \| x_n \|^2.$$

   (ii) Prove that every orthogonal set is LI. 5

9. (p) Let V be a finite dimensional inner product space. Then prove that V has an orthogonal (orthonormal) set as a basis. 5

   (q) Using Gram-Schmidt process, orthonormalise the set of vectors :

   $\{(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)\}$ of V_4. 5

UNIT—V

10. (a) If M_1 and M_2 are submodules of R-module M, then prove that $M_1 + M_2$ is a sub module of M. Moreover prove that $M_1 + M_2$ is a direct sum of M_1 and M_2 iff $M_1 \cap M_2 = \{0\}$. 5

    (b) Define :

    (i) R-module homomorphism

    (ii) Quotient module

    and prove that if A be a submodule of unital R module M, then prove that M/A is also unital R-module. 1+1+3

11. (p) If H and K are submodules of M then prove that $\frac{H+K}{K} \cong \frac{H}{H \cap K}$. 5

    (q) If T is a homomorphism of a R-module M to R-module H then prove that :

    (i) $T(0) = 0$

    (ii) $T(-m) = -T(m) \ \forall \ m \in M$

    (iii) $T(m_1 - m_2) = T(m_1) - T(m_2) \ \forall \ m_1 m_2 \in M$. 3

    (r) If M be an R-module and $m \in M$. Then prove that $A = \{rm / r \in R)$ is a submodule of M. 2

B.Sc. (Part-III) Semester-VI Examination

MATHEMATICS (New)

(Linear Algebra)

Paper—XI

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives : 10

(i) The basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} of the vector space $R^3(R)$ is known as :

(a) Normal basis (b) Quotient basis

(c) Standard basis (d) None of these

(ii) The vectors (a, b) and (c, a) are L.D iff :

(a) ad – bc = 0 (b) ab – cd = 0

(c) cd – ab = 0 (d) ab + dc = 0

(iii) The kernel of a linear transformation $T : U \rightarrow V$ is a subset of :

(a) U (b) V

(c) U and V (d) None of these

(iv) If W is a subspace of a finite dimensional vector space V, then dim (V/W) =

(a) $\frac{\dim V}{\dim W}$ (b) dim V – dim W

(c) dim V + dim W (d) None of these

(v) An element of dual space of V is called a :

(a) Linear functional (b) Bilinear element

(c) Linear element (d) None of these

(vi) Annihilator of W, A(W) is a subspace of :

(a) W (b) V

(c) $\hat{V}$ (d) None of these

(vii) Every set of orthogonal vectors is :

(a) Linearly Independent

(b) Linearly Dependent

(c) Linearly Independent and Linearly Dependent

(d) None of these

(viii) Let W be a subspace of an IPSV then $W \cap W^{\perp} =$

(a) $\{0\}$ (b) $\{1\}$

(c) ϕ (d) None of these

(ix) R-Module homomorphism is linear transformation if :

(a) R is with unit element (b) R is commutative

(c) R is a field (d) None of these

(x) If the ring R has a unit element 1 and 1.a = a for all $a \in M$, then M is called :

(a) A unital R-module (b) Right R-module

(c) Left R-module (d) None of these

UNIT—I

2. (a) Prove that intersection of two subspaces of a vector space is again a subspace. Is this statement is true for union ? 5

(b) Let U and W are two subspaces of a vector space V and Z = U + W. Then show that $Z = U \oplus W \Leftrightarrow z = u + w$ uniquely for any $z \in Z$ and for some $u \in U$ and $w \in W$. 5

3. (p) Define the Linear span of a subset of a vector space and show that Linear span L(S) of a subset S of a vector space V is the smallest subspace of V containing S. 5

(q) If U and W are finite dimensional subspaces of a vector space V, then prove that :

$$\dim(U + W) = \dim U + \dim W - \dim (U \cap W).$$ 5

UNIT—II

4. (a) Let $T : U \to V$ be a linear transformation. Then prove that :

   T is one-one $\Leftrightarrow N(T)$ is zero subspace of U. 5

   (b) Let $T : V_3 \to V_3$ defined by :

   $$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 - 2x_1)$$

   Find range, kernel, rank, nullity and verify rank-nullity theorem. 5

5. (p) State and prove Rank-Nullity theorem. 5

   (q) Find the transformation $T(x, y, z)$. If T is a linear map and matrix of T with respect to the bases B_1 and B_2 is $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$, where

   $B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and

   $B_2 = \{(1, 0), (2, -1)\}$. 5

UNIT—III

6. (a) Let U, V are finite dimensional complex vector spaces and $A : U \to V$, $B : U \to V$ be linear maps, $\alpha \in C$, then prove that :

   (i) $(A + B)^* = A^* + B^*$

   (ii) $(\alpha A)^* = \overline{\alpha} A^*$ 3+2

   (b) Prove that the element $\lambda \in f$ is a characteristic root of $T \in L(V)$ iff for some non zero $v \in V$, $Tv = \lambda v$. Also define characteristic root and characteristic vector. 3+1+1

7. (p) If V is a finite dimensional vector space over F, then prove that $V \approx \hat{V}$. 5

   (q) If W is a subspace of finite dimensional vector space V, then prove that $A(A(W)) = W$. 5

UNIT—IV

8. (a) Define inner product space and prove that in an inner product space V :

   (i) $\| \alpha \cdot u \| = | \alpha | \cdot \| u \|$

   (ii) $\| u + v \| \le \| u \| + \| v \|$, $\alpha \in F$ and $u, v \in V$. 1+4

   (b) Using Gram-Schmidt process orthonormalise the set of vectors $\{(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)\}$ of V_4. 5

9. (p) Prove that if $\{W_1, W_2, \ldots\ldots, W_m\}$ is an orthonormal set in V, then $\sum_{i=1}^{m} |(w_i, v)|^2 \leq \|v\|^2$ for any $v \in V$. 5

(q) Prove that every finite dimensional inner product space has an orthogonal basis. 5

UNIT—V

10. (a) Define Homomorphism of Modules and prove that if T is a homomorphism of an R-module M to an R-Module H, then :

(i) $T(0) = 0$

(ii) $T(-m) = -T(m) \; \forall \; m \in M$

(iii) $T(m_1 - m_2) = T(m_1) - T(m_2) \; \forall \; m_1 m_2 \in M$. 1+4

(b) Prove that every abelian group G is a module over a ring of integers Z. 5

11. (p) Define the sub module and prove that an arbitrary intersection of sub modules of a module is a submodule. 1+4

(q) Define direct sum of submodules and prove that if M_1 and M_2 are sub modules of R-module M, then $M_1 + M_2$ is a submodule of M. 1+4

B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS (New)

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The order of outer product is the _____ of the order of the tensors.

(a) Product (b) Difference

(c) Sum (d) None of these 1

(ii) The interval ds is said to be time like if :

(a) $ds^2 = 0$ (b) $ds^2 < 0$

(c) $ds^2 > 0$ (d) None of these 1

(iii) In Newtonian Mechanics, an event is identified by _____ real numbers.

(a) 1 (b) 2

(c) 3 (d) 4 1

(iv) 'Principle of Relativity' means :

(a) Some inertial frame are equivalent (b) All inertial frame are not equivalent

(c) All inertial frame are equivalent (d) None of these 1

(v) Length contraction means :

(a) Moving rod measures shorter (b) Moving rod measures larger

(c) Rest rod measures shorter (d) Rest rod measures longer 1

(vi) In relativistic addition law for velocities, when $c \to \infty$ then :

(a) $u' = u + v$ (b) $u' = u - v$

(c) $u' = v - u$ (d) None of these 1

(vii) Four velocity of a particle is defined as :

(a) $u^i = \frac{ds}{dx^i}$ (b) $u^i = \frac{dx^i}{ds}$

(c) $u = \frac{dx}{ds^i}$ (d) $u = \frac{dx^i}{ds}$ 1

(viii) $\overline{F}$ = mass X acceleration where mass = _______ is the longitudinal mass of the particle.

(a) $\frac{m_0}{(1 - u^2/c^2)^{1/2}}$ (b) $\frac{m_0}{(1 - u^2/c^2)^{3/2}}$

(c) $\frac{m_0}{(1 - u^2/c^2)^{-3/2}}$ (d) None of these 1

(ix) Mass energy equivalence relation is given by :

(a) $E = mc^2$ (b) $E = m/c^2$

(c) $E = c^2/m$ (d) None of these 1

(x) If $\overline{A}$ is a vector potential then magnetic field is given by :

(a) $\overline{H} = \text{div}\ \overline{A}$ (b) $\overline{H} = \text{curl}\ \overline{A}$

(c) $\overline{H} = \Delta\phi \times A$ (d) None of these 1

UNIT—I

2. (a) Obtain Galilean transformation equation for two inertial frames in relative motion. 4

(b) Show that the circle $x'^2 + y'^2 = a^2$ in s′ is measured to be an ellipse in s if s' moves with uniform velocity relative to s. 2

(c) Show that the Newton Kinematical equations of motion are invariant under Galilean transformation. 4

3. (a) What are Lorentz transformations ? Obtain an expression for them. 6

(b) Prove that in an inertial frame a body without influence of any forces, moves in a straight line with constant velocity. 4

UNIT—II

4. (a) Show that the velocities u and u′ measured in two inertial systems s and s′ are related by

$$\sqrt{1 - \frac{u^2}{c^2}} = \frac{\sqrt{1 - \frac{u'^2}{c^2}} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{u'_x v}{c^2}\right)}$$

where s′ is moving with velocity 'v' relative to s along xx′ axis. 5

(b) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

5. (a) Deduce the transformation of particle velocities and hence obtain relativistic addition law for velocities. 6

(b) Write a short note on 'Time dilation'. 4

UNIT—III

6. (a) Show that the interval or metric ds^2 between two events is given by :

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$$

Prove that ds^2 is invariant under Lorentz transformation. 6

(b) Prove that there exists an inertial system s′ in which the two events occur at one and the same point if the interval between two events is time like. 4

7. (a) Prove that :

(i) $T'^{14} = \alpha^2 \left\{ -\frac{v}{c} T^{11} + T^{14} + \frac{v^2}{c^2} T^{41} - \frac{v}{c} T^{44} \right\}$

(ii) $T'^{23} = T^{23}$. 6

(b) Define :

(i) Contravariant tensor of order one

(ii) Covariant tensor of order one

(iii) Kronecker delta

(iv) Space like interval. 4

UNIT—IV

8. (a) Prove that $E = mc^2$, where E is the energy of the particle. 6

(b) Show that the four velocity, in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1 - u^2/c^2}}, \frac{1}{\sqrt{1 - u^2/c^2}} \right), \text{ where } \bar{u} = (u_x, u_y, u_z).$$ 4

9. (a) Show that the quantity $p^2 - E^2/c^2$ is an invariant whose numerical value is $-m_0^2 c^2$. 4

(b) Define four momentum vector. Obtain the transformation equations for four momentum and energy. 6

UNIT—V

10. (a) Define current four vector. Transform its components under Lorentz transformation. Deduce an expression $c^2\rho^2 - J^2 = \rho_0^2 c^2 =$ invariant. 6

(b) Obtain the wave equation for the propagation of electric $\overline{E}$ and magnetic $\overline{H}$ field strengths in vacuum with velocity of light. 4

11. (a) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m_0^2 c^4 + c^2 \left(p + \frac{e}{e} A \right)^2 \right\}^{1/2} + e\phi.$$ 5

(b) Define electromagnetic field tensor F_{ij} and obtain the components F_{23}, F_{31}, F_{12}, also show that F_{ij} is antisymmetric. 5

AS–1473

B.Sc. (Part—III) Semester—VI Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **one** questions from each unit.

1. Choose the correct alternative : 1

(i) The reference system is said to be an inertial system if :

(a) Newton's first law of motion valid

(b) Newton's second law of motion valid

(c) Newton's third law of motion valid

(d) None of these.

(ii) The special Lorentz transformations will reduce to simple Galilean transformations when : 1

(a) V = C (b) V > > C

(c) V < < C (d) None of these

(iii) The simultaneity in special relativity is : 1

(a) relative (b) constant

(c) absolute (d) None of these

(iv) The time recorded by a clock moving with a body is known as : 1

(a) Time dilation (b) Proper time

(c) Fixed time (d) None of these

(v) The interval ds is said to be time-like if : 1

(a) $ds^2 = 0$ (b) $ds^2 < 0$

(c) $ds^2 > 0$ (d) None of these

(vi) Mass energy equivalence relation is given by : 1

(a) $E = mc^2$ (b) $E = m/c^2$

(c) $E = c^2/m$ (d) None of these

(vii) Four velocity of a particle is defined as :

(a) $u^i = \frac{ds}{dx^i}$ (b) $u^i = \frac{dx^i}{ds}$

(c) $u = \frac{dx}{ds^i}$ (d) $u = \frac{dx^i}{ds}$

(viii) If $\overline{A}$ is a vector potential then the magnetic field is given by : 1

(a) $\overline{H} = \text{div}\ \overline{A}$ (b) $\overline{H} = \text{curl}\ \overline{A}$

(c) $\overline{H} = \Delta\phi \times A$ (d) None of these

(ix) The electric and magnetic field strengths remain invariant under : 1

(a) Galilean transformations (b) Gauge transformations

(c) Fourier transformations (d) None of these

(x) The transformations $\overline{r}^1 = \overline{r} - \overline{v}t$ and $t^1 = t$ are known as : 1

(a) General Lorentz transformations (b) Special Lorentz transformations

(c) Simple Galilean transformations (d) General Galilean transformations

UNIT—I

2. (a) Discuss the Geometrical interpretations of Lorentz transformations. 4

(b) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(c) What are the postulates of the special theory of relativity ? 2

3. (a) Obtain Galilean transformation equations for two inertial frames in relative motion. 4

(b) Show that the Newton's kinematical equations of motion are invariant under Galilean transformations. 4

(c) Show that the circle $x'^2 + y'^2 = a^2$ in S' is measured to be an ellipse in S if S' moves with uniform velocity relative to S. 2

UNIT—II

4. (a) Deduce the transformations of particle velocities and hence obtain relativistic addition law for velocities. 6

(b) Obtain the transformation of the Lorentz contraction factor $\sqrt{1-u^2/c^2}$ 4

5. (a) If u and u' are the velocities of a particle measured in the frames S and S' respectively, then obtain the expressions a_x', a_y' and a_z' for acceleration of a particle. 6

(b) An observer moving along the x-axis of S with velocity V observes a body of proper volume V_0 moving with velocity u along the x axis of S. Show that the observer measures the volume to be equal to $V_0\sqrt{\frac{(c^2-v^2)(c^2-u^2)}{(c^2-uv)^2}}$. 4

UNIT—III

6. (a) Obtain the metric $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$ of the space time geometry of special relativity. Prove that ds^2 is invariant of special relativity. Prove that ds^2 is invariant under the Lorentz transformations. 5

(b) Define time-like and space-like intervals. Prove that there exists an inertial system S' in which two events occur at one and the same point if the interval between two events is time-like. 5

7. (a) Obtain the transformations of the components T'^{11} and T'^{14}. 6

(b) Define :

(i) Four dimensional radius vector

(ii) Four vector A^r

(iii) Light-like interval

(iv) World line. 4

UNIT—IV

8. (a) Deduce Einstein's mass-energy equivalence relation. 6

(b) A particle is given a kinetic energy equal to n times it's rest energy m_0c^2. What are :

(i) its speed and

(ii) momentum ? 4

9. (a) Prove that the mass of a moving particle with velocity u is $m = \frac{m_0}{\sqrt{1-u^2/c^2}}$, where m_0 is the mass of the particle when it is at rest. 6

(b) Show that four velocity and four acceleration are mutually orthogonal. 4

UNIT—V

10. (a) Obtain the transformations for electric and magnetic field strengths. 6

(b) Prove that the energy momentum tensor of electromagnetic field is trace free. 4

11. (a) Show that the Lorentz force acting on a particle of charge e is given by $\overline{F}_L = e\left(\overline{E} + \frac{1}{c}\overline{u} \times \overline{H}\right)$. 6

(b) Show that the Hamiltonian for a charged particle moving in an electromagnetic fields is :

$$H = \left\{m_0^2c^4 + c^2\left(P - \frac{e}{c}A\right)^2\right\}^{1/2} + e\phi.$$ 4

B.Sc. (Part-III) Semester-VI Examination

6S-MATHEMATICS

Special Theory of Relativity

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternatives :

(1) If $ds^2 = 0$, then the interval 'ds' is said to be : 1

(a) Light like (b) Space like

(c) Time like (d) None of these

(2) If an electromagnetic field is purely electric in an inertial frames, then the field in s' is : 1

(a) Only electric (b) Only magnetic

(c) Electric as well as magnetic (d) None of these

(3) If ϕ is a scalar potential and $\overline{A}$ is the vector potential, then the electric field is given by : 1

(a) $\overline{E} = \text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (b) $\overline{E} = \text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(c) $\overline{E} = -\text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (d) $\overline{E} = -\text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(4) The electric field strength $\overline{E}$ and the magnetic field strength $\overline{H}$ are invariant under : 1

(a) Galilean Transformations (b) Lorentz Transformations

(c) Gudge Transformations (d) None of these

(5) The time recorded by a clock moving with a body is called as : 1

(a) Absolute time (b) Proper time

(c) Improper time (d) None of these

(6) Length contraction means : 1

(a) Moving rod measures longer (b) Rest rod measures longer

(c) Moving rod measures shorter (d) Rest rod measures shorter

(7) Inertial system means the reference system where : 1

(a) Newton's first law of motion is valid

(b) Newton's second law of motion is valid

(c) Newton's third law of motion is valid

(d) None of these

(8) Four velocity of a particle is a : 1

(a) Unit space like vector (b) Unit time like vector

(c) Unit light like vector (d) None of these

(9) The simultaneity in special relativity is : 1

(a) Constant (b) Relative

(c) Absolute (d) None of these

(10) The interval ds is said to be space-like if : 1

(a) $ds^2 = 0$ (b) $ds^2 > 0$

(c) $ds^2 < 0$ (d) None of these

UNIT—I

2. (a) Prove that in an inertial frame a body without influence of any forces moves in a straight line with constant velocity. 3

(b) Show that Newton's kinematical equations of motion are invariant under Galilean transformations. 4

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 3

3. (p) Show that Lorentz transformation forms a group with respect to multiplication. 4

(q) Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 4

(r) What are the postulates of special relativity ? 2

UNIT—II

4. (a) Show that in nature no signal can move with a velocity greater than the velocity of light relative to any inertial system. 5

(b) Show that the velocities u and u' measured in two inertial systems and s' are related by :

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}} \cdot \sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{u'_x v}{c^2}\right)},$$

where s' is moving with velocity v relative to s along XX' axis. 5

5. (p) Obtain the transformations for the acceleration of a particle under special Lorentz transformations. 4

(q) Explain :

(i) Time Dilation

(ii) Length contraction. 3+3

UNIT—III

6. (a) Define four vector. Show that :

$A^1 = -A_1,\ A^2 = -A_2,\ A^3 = -A_3,\ A^4 = A_4.$ 4

(b) Prove that there exists an inertial system s' in which the two events occurs at one and the same time if the interval between two events is time like. 4

(c) What do you mean by covariant and contravariant tensor of rank two ? 2

7. (p) Define : Proper time. Show that the proper time of a moving object is always less than the corresponding interval in the rest system. 4

(q) Obtain the transformation of the components T'^{11} and T'^{12}. 4

(r) What are world points and world line ? 2

UNIT—IV

8. (a) Define four velocity. Prove that the four velocity in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-\frac{u^2}{c^2}}}, \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \right),$$

where $\bar{u} = (u_x, u_y, u_z) =$ velocity of the particle. 1+3

(b) Show that the quantity $p^2 - \frac{E^2}{c^2}$ is an invariant whose numerical value is $-m_0^2 c^2$. 3

(c) Prove that four velocity and four acceleration are mutually orthogonal. 3

9. (p) Obtain the mass energy equivalence relation $E = mc^2$, where m is the relativistic mass of the particle. 4

(q) Prove that $E = c\sqrt{p^2 + m_0^2 c^2}$ and $\frac{dE}{dp} = u$. 3

(r) Define four force. Show that the four force and the four velocity are orthogonal to each other. 1+2

UNIT—V

10. (a) Define current four vector. Show that $c^2\rho^2 - J^2$ is invariant and its value is $p_0^2.c^2$. 1+4

(b) Prove that the set of Maxwell's equations $\text{div}\,\overline{H} = 0$ and curl $\overline{E} = -\frac{1}{c}\frac{\partial \overline{H}}{\partial t}$ can be written as $\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j} = 0$, where F_{ij} is the electromagnetic field tensor. 5

11. (p) Define electromagnetic field tensor F_{ij}. Express the components of F_{ij} in terms of the electric and magnetic field strengths. 1+4

(q) Obtain the transformations for electric and magnetic field strengths. 5

B.Sc. (Part-III) Semester-VI Examination
MATHEMATICS (New)
(Special Theory of Relativity)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If ds = 0, then the interval ds is said to be : 1

(a) light like (b) space like

(c) time like (d) None of these

(ii) Lorentz transformation reduces to Galilean transformation if : 1

(a) V = C (b) V > > C

(c) V < < C (d) None of these

(iii) Signature of the Minkowskian space-time $ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$ is : 1

(a) 2 (b) –2

(c) 3 (d) 1

(iv) The transformations $\bar{r}' = \bar{r} - \bar{v}t$ and t' = t are known as : 1

(a) General Lorentz transformation (b) Special Lorentz transformation

(c) Simple Galilean transformation (d) General Galilean transformation

(v) The time recorded by a clock moving with a body is known as : 1

(a) Time dilation (b) Proper time

(c) Fixed line (d) None of these

(vi) The simultancity in special relativity is : 1

(a) relative (b) constant

(c) absolute (d) None of these

(vii) The four velocity of a particle is a unit ______ vector. 1

(a) space like (b) light like

(c) time like (d) None of these

(viii) Mass energy equivalence relation is given by : 1

(a) $E = mc^2$ (b) $E = m/c^2$

(c) $E = c^2/m$ (d) None of these

(ix) The scalar potential ϕ and vector potential A of the electric field is : 1

(a) $\overline{E} = \text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (b) $\overline{E} = \text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(c) $\overline{E} = -\text{grad}\,\phi - \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$ (d) $\overline{E} = -\text{grad}\,\phi + \frac{1}{c}\frac{\partial \overline{A}}{\partial t}$

(x) Four force f^i = ______ 1

(a) $\frac{du^i}{ds}$ (b) $\frac{dx^i}{ds}$

(c) $\frac{dp^i}{ds}$ (d) None of these

UNIT—I

2. (a) Define inertial system. Prove that in an inertial frame a body, without influence of any forces, moves in a straight line with constant velocity. 1+3

(b) Discuss the Geometrical interpretation of Lorentz transformation. 4

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 2

3. (p) Prove that Newton's fundamental equations of motion are invariant under the Galilean transformation. 4

(q) What are the postulates of special relativity ? 2

(r) Show that the three dimensional volume element dxdydz is not Lorentz invariant but the four dimensional volume elements dxdydzdt is Lorentz invariant. 4

UNIT—II

4. (a) Derive the transformation for the acceleration of a particle. Prove that when u, v < < c, these transformation deduce to Galilean one. 6

(b) Obtain the relativistic transformation formulae for the velocities of particle. 4

5. (p) Obtain the transformation of the Lorentz contraction factor $\sqrt{1-\frac{u^2}{c^2}}$. 6

(q) An observer moving along the x-axis of S with velocity V observes a body of proper volume V_0 moving with velocity u along the x-axis of S. Show that the observer measures the volume to be equal to $V_0\sqrt{\frac{(c^2-v^2)(c^2-u^2)}{(c^2-uv)^2}}$. 4

UNIT—III

6. (a) Define four tensor.

Prove that :

(i) $T'^{11}=\alpha^2\left\{T^{11}-\frac{V}{C}T^{14}-\frac{V}{C}T^{41}+\frac{V^2}{C^2}T^{44}\right\}$

(ii) $T'^{12}=\alpha\left\{T^{12}-\frac{V}{C}T^{42}\right\}$ 1+2+2

(b) Define length of four radius vector. Show that $x^1=-x_1$, $x^2=-x_2$, $x^3=-x_3$, $x^4=x_4$ and then deduce that $x_i=(-\bar{r}, ct)$. 1+3+1

7. (p) Define four vector A^r. Show that the square of the length of a four vector is invariant under Lorentz transformation. 1+4

(q) Prove that there exists an inertial system s' in which the two events occur at one and the same time if the interval between two events is space like. 5

UNIT—IV

8. (a) Prove that $L=-m_0c^2\sqrt{1-\frac{u^2}{c^2}}$ for the relativistic Lagrangian. 5

(b) Define four velocity. Prove that the four velocity, in component form can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c \cdot \sqrt{1 - \frac{u^2}{c^2}}}, \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \right),$$

where $\bar{u} = (u_x, u_y, u_z)$ = ordinary z-dimensional velocity of the particle. 5

9. (p) Obtain Einstein's mass energy equivalence relation. 5

(q) Define four velocity and four acceleration. Show that four velocity and four acceleration are mutually orthogonal. 2+3

UNIT—V

10. (a) Define electric and magnetic field strenths in terms of scalar ϕ and vector potential $\overline{A}$ and show that $\vec{E}$ and $\vec{H}$ remain invariant under Gauge transformation. 2+3

(b) Prove that the Lagrangian for a charge particle in electromagnetic field is :

$$L = -m_0 c^2 \sqrt{1 - \frac{u^2}{c^2}} + \frac{e}{c} \vec{A} \cdot \vec{u} - e\phi.$$ 5

11. (p) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is :

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\phi.$$ 5

(q) State Maxwell's equations of electromagnetic theory in vacuum. Also find its equations in component form. 2+3