B.Sc. (Part—III) Semester—V Examination
MATHEMATICS (New)
(Mathematical Analysis)
Paper—IX

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :

(i) Let f: [0, 1] → R be Riemann integrable. Which of the following is always true :

(a) f is continuous

(b) f is monotone

(c) f has only finite number of discontinuities

(d) the set of discontinuties of f may be infinite ? 1

(ii) An improper integral $\int_a^\infty \frac{dx}{x^p}$, a ∈ R is convergent if :

(a) p < 1 (b) p > 1

(c) p ≥ 1 (d) p = 1 1

(iii) β(m, n) is :

(a) $\lceil m \lceil n$ (b) $\frac{\lceil (m+n)}{\lceil m \lceil n}$

(c) $\frac{\lceil m \lceil n}{\lceil (m+n)}$ (d) $\frac{\lceil m \lceil n}{\lceil (m-n)}$ 1

(iv) In the real line R, which of the following is true ?

(a) Every bounded sequence converges (b) Every sequence converges

(c) Every Cauchy sequence converges (d) None of the above 1

(v) Every neighbourhood is a/an :

(a) Closed set (b) Open set

(c) Open closed set (d) None of the above 1

(vi) A function u(x, y) is harmonic in region D if :

(a) $u_{xx} - u_{yy} = 0$ (b) $u_{xy} + u_{yx} = 0$

(c) $u_{xy} - u_{yx} = 0$ (d) $u_{xx} + u_{yy} = 0$ 1

(vii) The function $f(z) = \sqrt{|xy|}$ is ______ at the origin.

(a) Harmonic function (b) Analytic function

(c) Conjugate function (d) Not analytic function 1

(viii) If f(z) and $\overline{f(z)}$ are both analytic functions then f(z) is :

(a) Identically zero (b) Constant

(c) Unbounded (d) None of the above 1

(ix) The points z where $|e^z| = 10$ form a :

(a) Circle (b) Straight line

(c) Hyperbola (d) Parabola 1

(x) A bilinear transformation with two non-infinite fixed points α and β having Normal form

$$\frac{w-\alpha}{w-\beta} = k\left(\frac{z-\alpha}{z-\beta}\right)$$ is Elliptic if :

(a) $|k| \neq 1$, k is real (b) $k \neq 1$, k is not real

(c) $|k| = 1$ (d) None of the above 1

UNIT—I

2. (a) Prove that every continuous function is integrable. 4

(b) Let the function f be defined as :

$$f(x) = 1, \quad \text{when } x \text{ is rational}$$
$$= -1, \quad \text{when } x \text{ is irrational}$$

Show that f is not R-integrable over [0, 1] but $|f| \in R\,[0, 1]$. 3

(c) Show that any constant function defined on a bounded closed interval is integrable. 3

3. (p) If f is a bounded and integrable function over [a, b] and M, m are bounds of f over [a, b], prove that :

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$ 4

(q) Prove that $\frac{2}{17} < \int_{-1}^{2} \frac{x}{1+x^4}\,dx < 1/2$. 3

(r) If f is continuous and non-negative on [a, b], then show that $\int_a^b f(x)\,dx \geq 0$. 3

UNIT—II

4. (a) Prove that the integral $\int_{a^+}^{b} \frac{dx}{(x-a)^p}$ converges if $p < 1$ and diverges if $p \geq 1$. 4

(b) Show that $\int_{1}^{\infty} \frac{\sin x}{x^2} dx$ converges absolutely. 3

(c) Show that $\int_{0}^{\infty} e^{-x^2} dx$ converges. 3

5. (p) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma (m+n)}$. 4

(q) Prove that $\int_{0}^{\pi/2} \sin^2\theta \cos^4\theta \, d\theta = \frac{\pi}{32}$. 3

(r) Prove that $\Gamma(n+1) = n\Gamma(n)$. 3

UNIT—III

6. (a) If $f(z) = u(x, y) + iv(x, y)$ be analytic in a region D, then prove that $u(x, y)$ and $v(x, y)$ satisfy Cauchy-Riemann equations. 4

(b) If $f(z)$ and $f(\overline{z})$ are analytic functions, prove that $f(z)$ is constant. 3

(c) Show that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate function. Hence find $f(z) = u + iv$. 3

7. (p) If u and v are harmonic in region R, prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)$ is analytic in R. 4

(q) If the function $f(z) = u + iv$ be analytic in domain D then prove that, the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are arbitrary constants. 3

(r) Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. 3

UNIT—IV

8. (a) Prove that, every bilinear transformation with two non infinite fixed points α, β is of the form $\frac{w-\alpha}{w-\beta} = k\frac{z-\alpha}{z-\beta}$, when k is constant. 5

(b) Under the transformation $w = \sqrt{2}e^{i\pi/4}z$, find the image of the rectangle bounded by $x = 0$, $y = 0$, $x = 2$ and $y = 3$. 5

9. (p) Prove that the cross ratio remains invariant under a bilinear transformation. 5

(q) Prove that under the transformation $w = \frac{z - i}{iz - 1}$ the region $I_m(z) \geq 0$ is mapped into the region $|w| \leq 1$. 5

UNIT—V

10. (a) Show that $d(x, y) = |x - y|, \forall x, y \in R$ defines a metric on $\mathbb{R}$. 5

(b) Define :

(i) Limit point

(ii) Boundary point. 2

(c) Prove that every neighbourhood is an open set. 3

11. (p) Define :

(i) Complete metric space

(ii) Open set. 2

(q) Prove that every convergent sequence in a metric space is a Cauchy sequence. 3

(r) Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$ then, prove that $d(x_n, y_n) \to d(x, y)$. 5

B.Sc. (Part—III) Semester—V Examination
5S : MATHEMATICS (New)
(Mathematical Methods)
Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No.**1** is compulsory and attempt it once.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternative (**1** mark each) :

(i) If $p_n(x)$ is the solution of Legendre's D.E., then $p_n(-1)$ is :

(a) –1 (b) 1

(c) $(-1)^n$ (d) 0

(ii) The value of integral $\int_{-1}^{1} x^2 p_1(x)\,dx$, where $p_1(x)$ is Legendre's polynomial of degree 1, equals :

(a) $\frac{2}{3}$ (b) $\frac{4}{35}$

(c) $\frac{4}{15}$ (d) 0

(iii) The value of $J_{1/2}(x)$ equals :

(a) $\sqrt{\frac{2}{n\pi}}\cos x$ (b) $\sqrt{\frac{2}{n\pi}}\sin x$

(c) $\sqrt{\frac{n\pi}{2}}\cos x$ (d) $\sqrt{\frac{n\pi}{2}}\sin x$

(iv) Eigen functions corresponding to different Eigen values are :

(a) Linearly dependent (b) Linearly independent

(c) Real (d) None

(v) The coefficient in a half range sine series for the function $f(x) = \sin x$ defined on $[0, \ell]$ is given by :

(a) $\int_0^{\ell} \sin x \cos \frac{n\pi x}{\ell}\,dx$ (b) $\int_0^{\ell} \cos x \cos \frac{n\pi x}{\ell}\,dx$

(c) $\frac{2}{\ell}\int_0^{\ell} \sin x \sin \frac{n\pi x}{\ell}\,dx$ (d) $\frac{2}{\ell}\int_0^{\ell} \sin x \sin \frac{n\pi x}{\ell}\,dx$

(vi) The function $f(x) = (-\sin x)^3$ is :

(a) Odd (b) Even

(c) Even and Odd (d) None of these

(vii) If $L[f(t)] = F(s)$, then $L[f(at)]$ is :

(a) $F(s - a)$ (b) $\frac{1}{a}F\left(\frac{s}{a}\right)$

(c) $F\left(\frac{s}{a}\right)$ (d) $aF\left(\frac{s}{a}\right)$

(viii) The value of $L^{-1}\left[\frac{1}{s-a}\right]$ is :

(a) 1 (b) t

(c) e^t (d) e^{at}

(ix) The Fourier sine transform of $f(x) = e^{|-x|}$, $x \geq 0$ is :

(a) $\frac{\lambda}{1+\lambda^2}$ (b) $\frac{\lambda}{1-\lambda^2}$

(c) $\frac{2\lambda}{1-\lambda^2}$ (d) $\frac{1}{1+\lambda^2}$

(x) If $F[f(x)] = F(\lambda)$, then the Fourier transform of $f(ax)$ is :

(a) $F\left(\frac{\lambda}{a}\right)$ (b) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right)$, $a = 0$

(c) $\frac{1}{|a|}F(\lambda)$ $a \neq 0$ (d) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right)$ $a \neq 0$ 10

UNIT—I

2. (a) Show that $p_n(x)$ is the coefficient of h^n in the ascending power series expansion of $(1 - 2xh + h^2)^{-1/2}$. 5

(b) Prove that $np_n = xp_n^1 - p_{n-1}^1$. 3

(c) Prove that $x^2 = \frac{1}{3}p_0(x) + \frac{2}{3}p_2(x)$. 2

3. (p) Prove that $\int_{-1}^{1}[p_x(x)]^2 dx = \frac{2}{2n+1}$. 5

(q) Prove that $p_x(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. 5

UNIT—II

4. (a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$. 4

(b) Prove that $xJ_p^1 = pJ_p - xJ_{p+1}$. 4

(c) Evaluate $\int_a^b J_0(x) \cdot J_1(x) dx$. 2

5. (p) Prove that Eigen values of the S-L problem are real. 4

(q) Prove that $\left(x^p \cdot J_p\right)' = x^p J_{p-1}$ 3

(r) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. 3

UNIT—III

6. (a) If the trigonometric series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges uniformly to f(x) in $c \le x < c + 2\pi$, then find the Fourier coefficient of f(x). 5

(b) Obtain Fourier Series in [0, 2] for the function $f(x) = x^2$. 5

7. (p) Obtain Fourier Series in $[-\pi, \pi]$ for the function :

$$f(x) = \begin{cases} -\pi & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}$$ 5

(q) Obtain Fourier cosine series in $[0, \pi]$ for the function f(x) = sin x. 5

UNIT—IV

8. (a) Prove that $L[t^n \cdot f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), n = 1,2,3.....$ 4

(b) Find L[sin t · cos 2t · cos 3t]. 3

(c) Show that $L(t^n) = \frac{n!}{s^{n+1}}, s > 0$. 3

9. (p) Solve the D.E. $y'' + 4y' = -8t$, $y(0) = y'(0) = 0$. 4

(q) Find the inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$ by using Convolution theorem. 3

(r) Prove that $L(u_{tt}) = s^2L(u(x, t)) - su(x, 0) - u_t(x, 0)$. 3

UNIT—V

10. (a) Find the finite Fourier sine and cosine transform of $f(x) = \sin ax$ in $(0, \pi)$. 4

(b) Find the Fourier transform of the function :

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$ 4

(c) Prove that $\int_0^{\ell} f'(x) \sin\frac{n\pi x}{\ell} dx = -\frac{n\pi}{\ell} F_c(n)$. 2

11. (p) Find the Fourier sine and cosine transform of the function $f(x) = x^{n-1}$, $n > 0$. 5

(q) Find finite Fourier cosine transform of u_x and u_{xx}; where $u = u(x, t)$. 5

B.Sc. (Part—III) Semester—V Examination

MATHEMATICS

Paper—IX

(Analysis)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :—

(i) $\int_{1}^{\infty} \frac{dx}{x^3}$ converges to : 1

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 3

(ii) If f be a bounded function defined on [a, b] and p be any partition of [a, b] then U(p, -f) is : 1

(a) L(p, f) (b) U(p, f)

(c) –L(p, f) (d) –U(p, f)

(iii) If f(z) and $f(\overline{z})$ are both analytic, then f(z) is : 1

(a) Unbounded (b) Constant

(c) Identically zero (d) None of these

(iv) A function F(x, y) is harmonic in D if : 1

(a) $F_{xx} + F_{yy} = 0$ (b) $F_{xx} - F_{yy} = 0$

(c) $F_{xy} + F_{yx} = 0$ (d) None of these

(v) If the transformation $w = \frac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into S, then S is : 1

(a) A circle (b) A straight line

(c) The region $R_e(w) \geq 0$ (d) The region $R_e(w) \leq 0$

(vi) A Bilinear transformation with only one fixed point is : 1

(a) Loxodromic (b) Elliptic

(c) Hyperbolic (d) Parabolic

(vii) If $\{A_\alpha\}$ be a finite or infinite collection of sets A_α then $\left[\bigcup_\alpha A_\alpha\right]^c =$ 1

(a) $\bigcap_\alpha A_\alpha^c$ (b) $\bigcup_\alpha A_\alpha^c$

(c) $\bigcap_\alpha A_\alpha$ (d) $\bigcup_\alpha A_\alpha$

(viii) In the real line R, which of the following is true ? 1

(a) Every Cauchy sequence is convergent

(b) Every sequence is bounded

(c) Every sequence is convergent

(d) None of these

(ix) A metric space (X, d) is complete if : 1

(a) Every convergent sequence in X is a Cauchy sequence

(b) Every Cauchy sequence in X is convergent in X

(c) Every convergent sequence in X is not a Cauchy sequence

(d) None of these

(x) If B is closed and K is compact, then $B \cap K$ is : 1

(a) Bounded (b) Closed

(c) Convergent (d) Compact

UNIT—I

2. (a) If f be continuous and integrable on [a, b] then prove that $\int_a^b f(x)\,dx = f(c)\,(b-a)$, where c is some point in [a, b]. 4

(b) If m and M are glb and lub of f(x) in [a, b] then show that

$$m(b-a) \le L(p, f) \le U(p, f) \le M(b-a).$$ 3

(c) If f is bounded function defined on [a, b] and p be any partition of [a, b] then prove that :

(i) $U(p, -f) = -L(p, f)$

(ii) $L(p, -f) = -U(p, f)$. 3

3. (p) Show that :

(i) $\int_0^\infty e^{-rx}\,dx$ converges if $r > 0$ and diverges if $r \le 0$. 3

(ii) $\int_a^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \le 1$ and $a > 0$. 3

(q) Using limit test, show that the integrals :

(i) $\int_2^\infty \frac{x}{1-x^2}\,dx = \infty$ and 2

(ii) $\int_1^\infty \frac{x\,dx}{3x^4+5x^2+1}$ coverges absolutely. 2

UNIT—II

4. (a) If $w = f(z) = u + iv$ be analytic in D and $z = re^{i\theta}$, where u, v, r, θ are the real numbers then prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. 5

(b) Separate sin z into real and imaginary parts. Use Cauchy-Riemann conditions to show that : sin z is analytic. Prove that $\frac{d}{dz}(\sin z) = \cos z$. 5

5. (p) Find an analytic function f(z) such that

$$R_e\{f'(z)\} = 3x^2 - 4y - 3y^2$$

and f(1 + i) = 0, using Milne-Thomson method. 5

(q) If f(z) = u + iv be analytic in the region D, where u and v have continuous partial derivatives upto the second order, then prove that u and v both are harmonic functions. 5

UNIT—III

6. (a) Prove that every bilinear transformation with two non-infinite fixed points α, β is of the form $\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$, where K is a constant. 5

(b) Find the fixed points of the bilinear transformation $w = \frac{(2+i)z-2}{i+z}$, what is its normal form ? Show that the transformation is Loxodromic. 5

7. (p) Find the image of the rectangle bounded by x = 0, y = 0, x = 2 and y = 3 under the transformation $w = \sqrt{2}\, e^{i\pi/4}.z$ 5

(q) Prove that the cross ratio remains invariant under a bilinear transformation. 5

UNIT—IV

8. (a) If X be a metric space with metric d then show that d_1 defined by

$d_1(x, y) = \frac{d(x, y)}{1+d(x, y)}$, is also a metric on x. 5

(b) If $\{x_n\}$ and $\{y_n\}$ are sequences in a metric space X such that $x_n \to x$ and $y_n \to y$. Then show that $d(x_n, y_n) \to d(x, y)$. 5

9. (p) Prove that the set A is open if and only if its complement is closed. 5

(q) Prove that the union of two nowhere dense sets in a metric space is nowhere dense. 5

UNIT—V

10. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. 6

(b) Let $f : R \to R$ such that

$$f(x) = \begin{cases} x & , \ x \text{ is irrational} \\ -x & , \ x \text{ is rational.} \end{cases}$$

Show that f is continuous only at $x = 0$. 4

11. (p) Let X, Y be metric spaces and $f : X \to Y$. Prove that f is continuous iff

$f^{-1}(B') \subseteq [f^{-1}(B)]'$ for every subset B of Y, $B' = \text{int } B$. 5

(q) If f be a continuous mapping of a connected metric space X into a metric space Y. Then prove that f(x) is connected. 5

UNIT—V

10. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X [illegible] and only if [illegible] open [illegible]

(b) [illegible] prove that [illegible]

[illegible] nominal [illegible]

[illegible] rational [illegible]

Show that the [illegible] continuous only at $x = 0$. [illegible]

11. (a) Let [illegible] metric spaces and [illegible] Prove that f is continuous [illegible]

[illegible] for every [illegible]

(b) If [illegible] continuous [illegible] compact [illegible]

[illegible] that [illegible] is continuous.

B.Sc. (Part-III) Semester-V Examination
MATHEMATICS (NEW)
Mathematical Analysis
Paper—IX

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :

(i) Consider P = (1, 2, 4) be a partition of interval [1, 4] then $\mu(P)$ is : 1

(a) 1 (b) 0

(c) 2 (d) 4

(ii) Let f be a bounded function defined on [a, b] and p be any partition of [a, b] then L(p, –f) is : 1

(a) –U(p, f) (b) –L(p, f)

(c) L(p, f) (d) U(p, f)

(iii) An integral $\int_0^\infty e^{-rx}\,dx$ is convergent if : 1

(a) $r < 0$ (b) $r > 0$

(c) $r = 0$ (d) None of these

(iv) The value of $\overline{|1/2}$ is : 1

(a) 1/2 (b) 1

(c) $\sqrt{\pi}$ (d) π

(v) If $f(z) = (x + ay) + i(bx + y)$ is analytic then : 1

(a) $a = b$ (b) $a + b = 0$

(c) $a = 1, b = 0$ (d) $a > b$

(vi) Let $f(z) = u + iv$ be analytic function and $z = re^{i\theta}$ then C-R equations are : 1

(a) $u_r = v_\theta$, $u_\theta = -v_r$

(b) $u_r = rv_\theta, u_\theta = -\frac{1}{r} v_r$

(c) $u_r = \frac{1}{r} v_\theta, v_r = -\frac{1}{r} u_\theta$

(d) $u_r = v_\theta$, $u_\theta = v_r$

(vii) A Mobius transformation which is not identity can have the following number of fixed points : 1

(a) 5

(b) 4

(c) 3

(d) 2

(viii) A bilinear transformation with two non-infinite fixed points p and q have normal form

$\frac{w-p}{w-q} = k\left(\frac{z-p}{z-q}\right)$ then BT is elliptic transformation if : 1

(a) $|k| = 1$

(b) $|k| \neq 1$

(c) $|k| = 0$

(d) $|k| = 2$

(ix) For any finite collection $A_1, A_2, \ldots A_n$ of open sets $\bigcap_{\alpha=1}^{n} A_\alpha$ is : 1

(a) Closed

(b) Open

(c) Semi open

(d) None of these

(x) Every neighbourhood of a point is : 1

(a) Closed

(b) Finite

(c) Open

(d) ϕ

UNIT—I

2\. (a) Let a bounded function f defined on [a, b] is integrable on [a, b] iff for each $\in > 0$ there exist a partition P of [a, b] such that $U(p, f) - L(p, f) < \in$. Prove this. 5

(b) Let the function f(x) be defined as $f(x) = \begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational.} \end{cases}$ Show that f is not R-integrable over [0, 1], but $|f| \in R[0, 1]$. 5

3. (a) If $f \in R[a, b]$, then prove that $F : [a, b] \to R$ defined by $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$. If f is continuous at $x_o \in [a, b]$, then prove that F is differentiable at x_o with $F'(x_o) = f(x_o)$. 5

(b) Prove that every continuous function is integrable. 5

UNIT—II

4. (a) Let $f(x), g(x) \in C$, $a \le x < \infty$ and $0 \le f(x) \le g(x)$, $\forall\, x \ge a$. Then prove that :

(i) $\int_a^\infty g(x)\,dx < \infty \Rightarrow \int_a^\infty f(x)\,dx < \infty$ and

(ii) $\int_a^\infty f(x)\,dx = \infty \Rightarrow \int_a^\infty g(x)\,dx = \infty$. 4

(b) Show that $\int_2^\infty \frac{x^2}{\sqrt{x^7+1}}\,dx$ is convergent. 3

(c) Show that $\int_1^\infty \frac{\sin x}{x^2}\,dx$ converges absolutely. 3

5. (a) Prove that :

$$\Gamma(1/2) = \sqrt{\pi}.$$ 4

(b) Evaluate :

$$\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx.$$ 3

(c) Show that :

$$\beta(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,dx.$$ 3

UNIT—III

6. (a) Prove that a necessary condition that $f(z) = u + iv$ be analytic in a region D is that $u_x = v_y$ and $u_y = -v_x$. 5

   (b) Show that the function $w = e^z$ is analytic function and find $\frac{dw}{dz}$. 5

7. (a) If the function $f(z) = u + iv$ is analytic in D then prove that families of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are constants. 4

   (b) If $f(z)$ and $f(\bar{z})$ are analytic functions then prove that $f(z)$ is constant. 3

   (c) Show that $w = e^{\bar{z}}$ is not analytic function for any z. 3

UNIT—IV

8. (a) Prove that the bilinear transformation is a combination of translation, rotation, stretching an inversion transformation. 5

   (b) Consider the transformation $w = ze^{i\pi/4}$ and determine the region in the w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. 5

9. (a) Prove that every bilinear transformation with single non-infinite fixed point α can be put in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + k$, where k is constant. 5

   (b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 5

UNIT—V

10. (a) Let the mapping $d : c[0, 1] \times c[0, 1] \to R$ be defined by $d(f, g) = \int_0^1 |f(x) - g(x)|\, dx$.

    Show that d is metric on $c[0, 1]$. 5

    (b) Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, then show that $d(x_n, y_n) \to d(x, y)$. 5

11. (a) Define neighbourhood of a point in a metric space X and prove that every neighbourhood of a point is open set. 5

    (b) Prove that every convergent sequence is Cauchy sequence and give an example of sequence which is Cauchy sequence but not convergent. 5

B.Sc. Part—III Semester—V Examination

MATHEMATICS (NEW)

(Mathematical Analysis)

Paper—IX

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :—

(i) If $P_1 = (1, 2, 4)$ and $P_2 = (1, 3, 4)$ be two partitions of [1, 4] then common refinement of P_1 and P_2 is : 1

(a) (1, 2, 4) (b) (1, 3, 4)

(c) (1, 4) (d) (1, 2, 3, 4)

(ii) Let F be bounded function defined on [a, b] and P be any partition of [a, b], if $\alpha < 0$ is any real number then $U(P, \alpha f)$ is : 1

(a) $\alpha L(P, f)$ (b) $\alpha U(P, f)$

(c) $U(P, f)$ (d) None of these

(iii) An improper integral $\int_{a+}^{b} \frac{1}{(x-a)^p} dx$ is divergent if : 1

(a) $p \geq 1$ (b) $p < 1$

(c) $p = \frac{1}{2}$ (d) None of these

(iv) $\beta(m, n)$ is : 1

(a) $\frac{\Gamma(m+n)}{\Gamma m \, \Gamma n}$ (b) $\frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$

(c) $\frac{\Gamma m \, \Gamma n}{\Gamma(m-n)}$ (d) $\Gamma m \, \Gamma n$

(v) A function u(x, y) is harmonic in D if : 1

(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} - u_{yy} = 0$

(c) $u_{xy} + u_{yx} = 0$ (d) $u_{xx} - u_{xy} = 0$

(vi) Let u, v be real valued function defined on $\mathbb{R}^2$ and $f(z) = u + iv$; $\bar{f}(z) = u - iv$. If f is an analytic function and f is not constant, then : 1

(a) $\bar{f}$ is always analytic (b) $\bar{f}$ may or may not be analytic

(c) $\bar{f}$ is never analytic (d) $f + \bar{f}$ is analytic

(vii) A bilinear transformation $w = \frac{az+b}{cz+d}$, is conformal if : 1

(a) $ad - bc = 0$ (b) $a \neq 0,\ b \neq 0$

(c) $ad - bc \neq 0$ (d) $c \neq 0,\ d \neq 0$

(viii) A bilinear transformation with two non-infinite fixed points α & β having Normal form

$\frac{w-\alpha}{w-\beta} = K\left(\frac{z-\alpha}{z-\beta}\right)$ is Hyperbolic if : 1

(a) $|K| = 1$ (b) $|K| \neq 1$, K is real

(c) $|K| \neq 1$, K is not real (d) None of these

(ix) Let (X, d) be metric space and $A \subset X$, A is nonempty the diameter of A is d(A) if A is unbounded then : 1

(a) $d(A) < \infty$ (b) $d(A) = -\infty$

(c) $d(A) = \infty$ (d) $d(A) = 1$

(x) Let A be a nonempty closed subset of metric space (X, d) then A^C is : 1

(a) open (b) closed

(c) ϕ (d) None of these.

UNIT—I

2. (a) Prove that if f(x) is monotonic function in.[a, b] then it is integrable on [a, b]. 4

(b) If f, g $\in R[a, b]$ and $f(x) \leq g(x)$, $\forall x \in [a, b]$, then prove that $\int_a^b f(x)dx \leq \int_a^b g(x)dx$. 3

(c) Show that any constant function defined on [a, b] is integrable on [a, b]. 3

3. (a) If a function F(x) is continuous on [a, b] and F(x) is continuous and differentiable on [a, b] with $F'(x) = F(x)$, $x \in [a, b]$, then prove that $\int_a^b F(x)dx = F(b) - F(a)$. 4

(b) Let f(x) be a bounded function defined on [a, b] with bounds m and M. Then prove that : $m(b - a) \le L(P, f) \le U(P, f) \le M(b - a)$ for any partition P of [a, b]. 3

(c) Define Darboux Upper and Lowers sums for bounded function f(x) defined on [a, b] and find them for function f(x) with bounds $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, $m_4 = 4$ and $M_1 = 2$, $M_2 = 3$, $M_3 = 4$, $M_4 = 5$ for the partition P = {1, 3, 4, 5, 6} of [1, 6]. 3

UNIT—II

4. (a) Prove that $\int_a^\infty \frac{1}{x^p} dx$ converges if $p > 1$ and diverges if $p \le 1$ and $a > 0$. 4

(b) Show that $\int_2^\infty \frac{x^3}{\sqrt{x^7 + 1}} dx$ is divergent. 3

(c) Show that $\int_2^\infty \frac{\cos x}{\sqrt{1 + x^3}} dx$ is Absolutely convergent. 3

5. (a) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma (m + n)}$. 4

(b) Show that $\int_0^1 \sqrt{x(1 - x)}\, dx = \pi/8$. 3

(c) Prove that $\Gamma n = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$. 3

UNIT—III

6. (a) If $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic function in D, then prove that $u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u\theta$, CR equations in polar coordinates. 5

(b) Using Milne-Thomson method construct analytic function f(z), whose real part is $e^{-x}(x \cos y + y \sin y)$. 5

7. (a) Let $f(z) = u + iv$ be analytic in the region D, where u and v have continuous partial derivatives upto the second order. Then prove that u and v are harmonic functions. 5

(b) If $w = u + iv$ is analytic function in the region R, then prove that $\frac{\partial(u, v)}{\partial(x, y)} = \left|f^1(z)\right|^2$. 3

(c) If $w = u + iv$ is analytic function in D, then prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x}$. 2

UNIT—IV

8. (a) Prove that the cross-ratio remains invariant under bilinear transformation. 5

(b) Find the image of the rectangle bounded by $x = 0$, $y = 0$, $x = 2$ and $y = 3$ under the transformation $w = e^{i\pi/4} \times \sqrt{2}$. 5

9. (a) Prove that every bilinear transformation with single non-infinite fixed point α can be put in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + K$, where K is a constant. 5

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into points $w = i, o, -i$. 5

UNIT—V

10. (a) Let X be an arbitrary non-empty set. Define d by $d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$ show that 'd' is metric on X. 5

(b) Let (X, d) be a metric space and $x, y, x', y' \in X$. Show that

$$|d(x, y) - d(x', y')| \leq d(x, x') + d(y, y').$$ 3

(c) Define :—

(i) Limit point

(ii) Interior point of a set A. 2

11. (a) Let Y be a subspace of a complete metric space X. Then prove that Y is complete $\Leftrightarrow$ Y is closed. 5

(b) Prove that every neighborhood of a point is open set. 3

(c) Define :—

(i) Cauchy sequence

(ii) Complete metric space. 2

B.Sc. (Part—III) Semester—V Examination
MATHEMATICS (NEW)
Paper—IX
(Mathematical Analysis)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) Let P = (1, 3, 4, 5, 6) be a partition of [1, 6] and if $M_1 = 2$, $M_2 = 3$, $M_3 = 4$, $M_4 = 5$ are lub's of F then U(P, F) is : 1

(a) 11 (b) 10

(c) 12 (d) 16

(ii) Let f be a bounded function defined on [a, b] and P be any partition of [a, b], P* be refinement of P. Then L(P, f) and L(P*, f) satisfy : 1

(a) $L(P, f) \le L(P^*, f)$ (b) $L(P, f) \ge U(P^*, f)$

(c) $L(P, f) \ge L(P^*, f)$ (d) None of these

(iii) An improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ converges to : 1

(a) $\sqrt{x}$ (b) $-x$

(c) x (d) 0

(iv) An integral $\int_0^{\infty} e^{-kx} x^{n-1} dx$ is : 1

(a) $k^n \overline{|n}$ (b) $\frac{\overline{|n}}{k^n}$

(c) k^n (d) $\overline{|n}$

(v) If a function f(z) = u(x, y) + iv(x, y) is Analytic in a region D, then : 1

(a) $u_x = u_y$ and $u_y = v_x$ (b) $u_x = -u_y$ and $u_y = -v_x$

(c) $u_x = v_y$ and $u_y = -v_x$ (d) None of these

(vi) If $w = u + iv$ is analytic function in D, then $\frac{dw}{dz}$ is : 1

(a) $-\frac{\partial w}{\partial x}$ (b) $\frac{\partial w}{\partial y}$

(c) $-\frac{\partial w}{\partial y}$ (d) $\frac{\partial w}{\partial x}$

(vii) A Mobius transformation $w = az$, a is real number, is : 1

(a) Rotation transformation (b) Magnification transformation

(c) Translation transformation (d) None of these

(viii) A bilinear transformation with only one fixed point is : 1

(a) Loxodromic (b) Parabolic

(c) Elliptic (d) Hyperbolic

(ix) For any collection of $\{A_\alpha\}$ open sets, $\bigcup_\alpha A_\alpha$ is : 1

(a) Closed (b) Open

(c) Semi-open (d) None of these

(x) A metric space (X, d) is complete if every Cauchy sequence in X is : 1

(a) Bounded (b) Unbounded

(c) Convergent (d) Divergent

UNIT—I

2. (a) Prove that a bounded function f defined on [a, b] is integrable on [a, b] iff for any $\epsilon > 0$ there exist a $\delta > 0$ such that for every partition P of [a, b] with $\mu(P) < \delta$, $U(P, f) - L(P, f) < \epsilon$. 5

(b) If f is function defined by $f(x) = x$ on [0, 2], then show that f is integrable in Riemann sense over [0, 2] and $\int_0^2 f(x)\,dx = 2$. 5

3. (a) Prove that if f is continuous and integrable on [a, b], then $\int_a^b f(x)\,dx = f(c)(b-a)$ where c is some point in [a, b]. 5

(b) Let the function f be defined as $f(x) = \begin{cases} 1 & , x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$. Show that f is not R-integrable on [0, 1]. But $|f| \in R[0, 1]$. 5

UNIT—II

4. (a) Prove that $\int_{a+}^{b} \frac{1}{(x-a)^p} dx$ converges if $p < 1$ and diverges if $p \geq 1$. 4

(b) Test the convergence of $\int_{2}^{\infty} \frac{1}{\sqrt{x^2-1}} dx$. 3

(c) Show that $\int_{1}^{\infty} \frac{e^{-x}}{x} dx$ is convergent. 3

5. (a) Prove that :

$$\beta(m, n) = \int_{0}^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_{0}^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx.$$ 4

(b) Evaluate :

$$\int_{0}^{\infty} \sqrt{x}\, e^{-\sqrt[3]{x}} dx.$$ 3

(c) Prove that :

$$\int_{0}^{\infty} e^{-kx} x^{n-1} dx = \Gamma(n) / k^n.$$ 3

UNIT—III

6. (a) Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic function in region D, then $u_x = v_y$ and $u_y = -v_x$ in D. 5

(b) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Find its conjugate and the corresponding analytic function f(z) in terms of z. 5

7. (a) If the function $f(z) = u + iv$ is analytic in the domain D, then prove that family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are constants. 5

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although C-R equations are satisfied at origin. 5

UNIT—IV

8. (a) Prove that cross-ratio remains invariant under a bilinear transformation. 5

(b) Let D be a region in z-plane is bounded by $x = 0$, $y = 0$, $x = 2$ and $y = 1$. Find the region in w-plane into which D is mapped under the transformation $w = z + (1 - 2i)$. 5

9. (a) Prove that every bilinear transformation with two non-infinite fixed points p, q can be put in the normal form $\frac{w-p}{w-q}=k\left(\frac{z-p}{z-q}\right)$ where k is constant. 5

(b) Find the fixed points of the transformation $w=\frac{z-1}{z+1}$; state whether it is elliptic, hyperbolic or Loxodromic. Find also its normal form. 5

UNIT—V

10. (a) If p is a limit point of a set A, then prove that every neighbourhood of p contains infinitely many points of A. 4

(b) Show that $d(x, y) = |x - y|, \forall x, y \in R$ defines a metric on R. 4

(c) Define :

(i) Open set

(ii) Closed set. 2

11. (a) Define a Cauchy sequence in a metric space and prove that every convergent sequence in a metric space is a Cauchy sequence. 4

(b) Prove that for any finite collection $A_1, \ldots, A_n$ of open sets $\bigcap_{i=1}^{n} A_i$ is open set. 4

(c) Define a metric d on a space X. 2

B.Sc. (Part-III) Semester-V Examination

MATHEMATICS (NEW)

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once and solve **ONE** question from each unit.

1. Choose correct alternative (**1** mark each) : 10

(1) The value of $P_n'(1)$ is :

(a) n (b) n + 1

(c) n(n + 1) (d) $\frac{1}{2}n(n+1)$

(2) If $P_n(x) = x$, then the value of n is :

(a) 0 (b) –1

(c) 1 (d) None

(3) The value of $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2$ is :

(a) $\frac{2}{\pi x}$ (b) $\frac{\pi x}{2}$

(c) $\frac{\pi}{2}$ (d) Zero

(4) Each eigen function $y_n(x)$ corresponding to the eigen values λ_n (n = 1, 2, ...) has exactly ______ zeros in (a, b).

(a) n – 1 (b) n

(c) n + 1 (d) One

(5) The function cos x has period :

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) None

(6) If the Fourier series correspond to an odd function f(x) in [–L, L], then its expansion contains only :

(a) constant terms (b) cosine terms

(c) sine terms (d) All above

(7) If $L[f(t)] = \frac{3}{s^2+9}$ for s > 0, then f(t) is :

(a) sin t (b) sin 2t

(c) sin 3t (d) None

(8) The inverse Laplace transform of $\frac{1}{s^2+a^2}$ is :

(a) sin at (b) $\frac{1}{a}$ sin at

(c) $\frac{1}{a}$ sin t (d) sin t

(9) Shifting property of the Fourier transform is :

(a) $F[f(x - a)] = F(\lambda)$ (b) $F[f(x - a)] = F\left(\frac{\lambda}{a}\right)$

(c) $F[f(x - a)] = e^{\lambda a} F(a)$ (d) $F[f(x - a)] = e^{-i\lambda a} \cdot F(\lambda)$

(10) The Fourier transform of f(x) . cos ax is :

(a) $F(\lambda + a) + F(\lambda - a)$ (b) $\frac{1}{2}[F(\lambda + a) + F(\lambda - a)]$

(c) $\frac{1}{2}[F(\lambda + a) - F(\lambda - a)]$ (d) None

UNIT—I

2. (a) Prove that :

$$\int_{-1}^{1} [P_n(x)]^2 \, dx = \frac{2}{2n+1} \text{ if } m = n.$$ 5

(b) Use Rodrigues formula to find $P_n(x)$, n = 0, 1, 2, 3, 4. 5

3. (p) Prove that :

$$(2n + 1) \, xP_n = (n + 1)P_{n+1} + nP_{n-1}.$$ 5

(q) Prove that :

$$\int_{-1}^{1} (x^2 - 1) P_n' \; P_{n+1} \, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}.$$ 5

UNIT—II

4. (a) Prove that :

$$\frac{d}{dx}[J_p(x)] = \frac{1}{2}[J_{p-1}(x) - J_{p+1}(x)].$$ 5

(b) Prove that :

$$xJ_p' = -p \, J_p + x \, J_{p-1}.$$ 5

5. (p) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5

(q) Prove that the eigen values of SL problem are real. 5

UNIT—III

6. (a) Obtain the Fourier series for $f(x) = | x |$ in $(-\pi, \pi)$. Hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots .$$ 5

(b) Expand $f(x) = 2x - x^2$ in the range (0, 3) as a Fourier series with period 3. 5

7. (p) Obtain the Fourier cosine series for $f(x) = x^2$ in $0 < x < 2$. 5

(q) Obtain the Fourier series for $f(x) = \cos\frac{x}{2}$ in $-\pi \le x \le \pi$. 5

UNIT—IV

8. (a) Find :

$L[\cosh^4 t]$. 3

(b) Use the transformations of derivatives to find the Laplace transform of cos at. 3

(c) If $L[f(t)] = F(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$, provided the integral exists. 4

9. (p) Find the inverse Laplace transform of $\frac{s}{s^4 - a^4}$. 3

(q) Find the inverse Laplace transform of $\frac{1}{(s-2)(s+2)^2}$ by convolution theorem. 3

(r) Use Laplace transform to solve the equation :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t},\ y(0) = 4,\ y'(0) = 2.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of $f(x) = x^2$, $0 < x < l$. 5

(b) Find the Fourier sine and cosine transforms of x^{n-1}, $n > 0$. 5

11. (p) Find Fourier sine transform of $f(x) = e^{-|x|}$, $x \geq 0$. Hence show that :

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m},\ m > 0.$$ 5

(q) Show that finite Fourier sine and cosine transforms and their inverses are all Linear transformations. 5

AU–130

B.Sc. Part—III (Semester—V) Examination
5S : MATHEMATICS (New)
(Mathematical Methods)
Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it **once** and solve **one** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) If $P_n(x) = (-1)^n$, then what is the value of x ?

(a) 1 (b) –1

(c) 0 (d) None

(ii) All roots of $P_n(x) = 0$ are :

(a) Distinct (b) Equal

(c) Complex (d) None

(iii) What is the value of $J_{-1/2}\left(\frac{\pi}{2}\right)$?

(a) –1 (b) 1

(c) 0 (d) π

(iv) Eigen functions corresponding to different eigen values are :

(a) Linearly dependent (b) Linearly independent

(c) Real (d) None

(v) The fundamental period of tan x is :

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) None

(vi) Fourier series are associated with :

(a) Algebraic functions

(b) Special functions

(c) Periodic functions defined on some interval I

(d) Linear functions

(vii) The inverse Laplace transform of $\frac{1}{s-a}$ is :

(a) 1 (b) t

(c) e^t (d) e^{at}

(viii) The Laplace transform of cost is :

(a) $\frac{1}{s^2+1}$ (b) $\frac{s}{s^2+1}$

(c) $\frac{1}{s^2-1}$ (d) $\frac{s}{s^2-1}$

(ix) If $F[f(x)] = F(\lambda)$, then the Fourier transform of f(ax) is :

(a) $F\left(\frac{\lambda}{a}\right)$ (b) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right), a = 0$

(c) $\frac{1}{|a|}F(\lambda), a \neq 0$ (d) $\frac{1}{|a|}F\left(\frac{\lambda}{a}\right), a \neq 0$

(x) The Fourier transform of convolution of f(x) and g(x) for $-\infty < x < \infty$ is :

(a) $F[f * g] = F[f(x)] \cdot F[g(x)]$ (b) $F[f * g] = F[f(x)] + F[g(x)]$

(c) $F[f * g] = F[f(x)] - F[g(x)]$ (d) $F[f * g] = F[f(x)] / F[g(x)]$ 10

UNIT—I

2. (a) Find $P_3(x)$ by Rodrigues formula and show that $\int_{-1}^{1} x^3 P_3(x)\,dx = \frac{4}{35}$. 2+3

(b) Show that $\int_{-1}^{1} [P_n'(x)]^2\,dx = n(n+1)$. 5

3. (p) Prove that $nP_n = xP_n' - P_{n-1}'$, where $P_n' = \frac{dP_n}{dx}$. 5

(q) Show that $\int_{-1}^{1} x^m P_n(x)\,dx = 0$ if $m < n$. 5

UNIT—II

4. (a) Prove that $xJ'_p = -pJ_p + xJ_{p-1}$. 5

(b) Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$. 5

5. (p) Prove that $J_p(-x) = (-1)^p J_p(x)$, if p is an integer. 5

(q) Prove that the eigen values of the SL problem are real. 5

UNIT—III

6. (a) Obtain the Fourier series for $f(x) = x \cos x$ in $[-\pi, \pi]$. 5

(b) Obtain the Fourier sine series for $f(x) = x^2$ in $0 < x < 2$. 5

7. (p) Express the function $f(x) = \pi x - x^2$ as Fourier sine series in $0 \le x \le \pi$. Deduce that :

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}.$$ 5

(q) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. 5

UNIT—IV

8. (a) Find $L[t^n]$, where n is a positive integer. 3

(b) Using transformations of derivatives find $L[t \sin at]$. 3

(c) Find the Laplace transform of $\int_0^t \frac{e^t \sin t}{t} dt$. 4

9. (p) Find $L^{-1}\left[\frac{s^2 - 3s + 4}{s^3}\right]$. 3

(q) Find the inverse Laplace transform of $\frac{1}{s(s^2+4)}$ by the convolution theorem. 3

(r) Use Laplace transform to find the solution of the equation :

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \; x(0) = 1, \; x\left(\frac{\pi}{2}\right) = -1.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of $f(x) = e^{ax}$ in $(0, \ell)$. 5

(b) Find $F\left[e^{-|x|}\right]$ and hence show that :

$$F\left[e^{-|2x|}\right] = \frac{4}{4+\lambda^2}.$$ 5

11. (p) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$, $a > 0$. Hence evaluate $\int_0^\infty \tan^{-1}\frac{x}{a} \cdot \sin x \, dx$. 5

(q) Find the Fourier sine and cosine transforms of $f(x) = x^n e^{-ax}$, $n > 0$. 5

B.Sc. (Part—III) Semester—V Examination
5S : MATHEMATICS (New)
(Mathematical Methods)
Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) If $p_n(x) = 1$, then what is the value of n ?

(a) 1 (b) –1

(c) 0 (d) None

(ii) The integral $\int_{-1}^{1} p_n(x) \cdot p_m(x)\, dx \neq 0$ if :

(a) $m < n$ (b) $m > n$

(c) $m \neq n$ (d) $m = n$

(iii) What is the value of $J_{1/2}\left(\frac{\pi}{2}\right)$?

(a) 0 (b) 1

(c) π (d) $\frac{\pi}{2}$

(iv) The eigen values of Strum-Liouville problem are :

(a) Real (b) Complex

(c) Equal (d) None

(v) Every Fourier series is a :

(a) Trigonometric series (b) Power series

(c) Exponential series (d) None

(vi) The fundamental period of sin x is :

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) None

(vii) If $L[f(t)] = \frac{1}{s^2}, (s > 0)$, then f(t) is :

(a) t^n (b) t^2

(c) 1 (d) t

(viii) Every bounded function is of exponential order :

(a) 1 (b) –1

(c) 0 (d) 2

(ix) If $F[f(x)] = F(\lambda)$, then Fourier transform of $f(x-a)$ is :

(a) $e^{\lambda a} \cdot F(\lambda)$ (b) $e^{i\lambda a} \cdot F(\lambda)$

(c) $e^{-i\lambda a} \cdot F(\lambda)$ (d) None

(x) The Fourier transform of $e^{-|x|}$ is :

(a) $\frac{2}{1+\lambda^2}$ (b) $\frac{1}{1+\lambda^2}$

(c) $\frac{2}{1-\lambda^2}$ (d) $\frac{1}{1-\lambda^2}$ 10

UNIT—I

2\. (a) Show that $p_n(1) = 1$ and $p_n(-x) = (-1)^n p_n(x)$. Hence or otherwise deduce that $p_n(-1) = (-1)^n$. 5

(b) Prove that $(2n+1)x\,p_n = (n+1)p_{n+1} + n\,p_{n-1}$. 5

3\. (p) Prove that :

(i) $\int_{-1}^{1} p_n(x)\,dx = 0,\ n \neq 0$ 4

(ii) $\int_{-1}^{1} p_0(x)\,dx = 2$. 1

(q) Show that $\int_{-1}^{1} p_m(x) \cdot p_n(x)\,dx = 0$ if $m \neq n$. 5

UNIT—II

4\. (a) Prove that $xJ'_p = pJ_p - xJ_{p+1}$. 5

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5

5\. (p) Prove that $J_{-p}(x) = (-1)^p J_p(x)$, if p is a positive integer. 5

(q) Find all the eigen values and eigen functions of the SL problem $y'' + \lambda^2 y = 0,\ y'(0) = y'(\ell) = 0$, $0 \leq x \leq \ell$. 5

UNIT—III

6\. (a) Obtain the Fourier series for $f(x) = x^2$ in $[-\pi, \pi]$. Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ 5

(b) Find the Fourier series for the function f(x) defined in $-\pi < x < \pi$ as :

$$f(x) = \begin{cases} -x & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}.$$

Deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 5

7. (p) Obtain the Fourier series for $\sqrt{1 - \cos x}$ in $(0, 2\pi)$. Hence deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{(4n^2 - 1)}$. 5

(q) Obtain the cosine half range series for the function $f(x) = x$ in $0 < x < 2$. 5

UNIT—IV

8. (a) If $L[f(t)] = F(s)$, then prove that $L[e^{at} f(t)] = F(s - a)$. 3

(b) Find the Laplace transform of $t^2 \sin at$. 3

(c) Evaluate $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$. 4

9. (p) Find the inverse Laplace transform of $\frac{6s - 4}{s^2 - 4s + 20}$. 3

(q) Verify the convolution theorem for $f_1(t) = t$, $f_2(t) = \cosh t$. 3

(r) Using Laplace transform method, solve the equation :

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}, \; y(0) = 4, \; y'(0) = 2.$$ 4

UNIT—V

10. (a) Find the finite Fourier sine and cosine transforms of mx, $0 < x < \ell$. 5

(b) Show that Fourier cosine transform of $f(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}} e^{-\lambda^2/4}$. 5

11. (p) Find the Fourier sine and cosine transforms of x^{n-1}, $n > 0$. 5

(q) Find the finite Fourier sine and cosine transforms of $f(x) = \sin ax$ in $(0, \pi)$. 5