B.Sc. Part-II (Semester-IV) Examination
MATHEMATICS
(Classical Mechanics)
Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.
(2) Solve **ONE** question from each unit.

1. Choose the correct alternative :

(i) Each planet describes ______ having the sun at one of its foci. 1

(a) An ellipse (b) A circle
(c) A hyperbola (d) None of these

(ii) If a bead is sliding along the wire then the constraint is ______. 1

(a) Holonomic (b) Non-holonomic
(c) Superfluous (d) None of these

(iii) For an inverse square law, the virial theorem reduces to ______. 1

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = n\overline{V}$
(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -\overline{V}$

(iv) The virtual work on a mechanical system by the applied forces and reversed effective forces is ______. 1

(a) Zero (b) One
(c) Negative (d) None of these

(v) The shortest distance between two points in a space is ______. 1

(a) A circle (b) A straight line
(c) An ellipse (d) A parabola

(vi) If H is the Hamiltonian of the system then a generalized coordinate q_i is said to be cyclic if ______. 1

(a) $\frac{\partial H}{\partial q_i} \neq 0$ (b) $\frac{\partial H}{\partial q_i} > 0$
(c) $\frac{\partial H}{\partial q_i} = 0$ (d) $\frac{\partial H}{\partial q_i} < 0$

(vii) A square matrix A is said to be orthogonal if ______. 1

(a) $A = A^T$ (b) $A^T = A^{-1}$
(c) $A = A^{-1}$ (d) None of these

(viii) The general displacement of a rigid body with ______ point fixed is a rotation about some axis. 1

(a) One (b) Two

(c) Three (d) None of these

(ix) The sum of the finite rotations depends on the ______ of the rotation. 1

(a) Degree (b) Order

(c) Both Degree and Order (d) None of these

(x) A particle moving in a space has ______ degrees of freedom. 1

(a) One (b) Two

(c) Three (d) Four

UNIT—I

2. (a) Derive the Lagrange's equations of motion in the form :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q_i}=0, i=1,2,........,n.$$

for conservative system from D'Alembert's principle. 6

(b) A bead is sliding on a uniformly rotating wire in a force-free space, then show that the acceleration of the bead is $\ddot{r}=rw^2$, where w is the angular velocity of rotation. 4

3. (p) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particles is $\left\{\frac{(m_1-m_2)}{(m_1+m_2)}\right\}^g$. 5

(q) Obtain the equations of motion of a simple pendulum by using D'Alembert's principle. 5

UNIT—II

4. (a) For a central force field, show that Kepler's second law is a consequence of the conservation of angular momentum. 5

(b) Prove that if the potential energy is a homogeneous function of degree –1 in the radius vector $\vec{r}_i$, then the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 5

5. (p) Prove that in a central force field the areal velocity is conserved. 5

(q) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle, then the force varies as the inverse fifth power of the distance. 5

UNIT—III

6. (a) Show that the functional :

$$I[y(x)] = \int_0^1 \{2y(x) + y'(x)\}\, dx$$

defined in the space $c_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

(b) Find the extremals of $I[y(x)] = \int_a^b [y^2 + y'^2 + 2ye^x]dx$. 5

7. (p) Find the extremals of the functional :

$$I[y(x)] = \int_a^b [16y^2 - y''^2 + x^2]dx.$$ 5

(q) Write down the Euler-Ostrogradsky equation for the functional :

$$I[z(x, y)] = \iint_D \left\{\left(\frac{\partial z}{\partial x}\right)^4 + \left(\frac{\partial z}{\partial y}\right)^4 + 12zf(x, y)\right\} dx\, dy.$$ 5

UNIT—IV

8. (a) Show that Hamilton's principle can be derived from D'Alembert's principle. 5

(b) Define Hamiltonian H. Derive the Hamilton's equations for the Hamiltonian H of the system. 1+4

9. (p) Deduce the Hamilton's equations of motion of a particle of mass m in Cartesian coordinates (x, y, z). 5

(q) Define Routhian, prove that a cyclic coordinate will not occur in the Routhian R. 1+4

UNIT—V

10. (a) Prove that if A is any 2×2 orthogonal matrix with determinant $|A| = 1$, then A is a rotation matrix. 5

(b) Define infinitesimal rotation. Prove that infinitesimal rotations commute. 1+4

11. (p) Show that two complex eigenvalues of an orthogonal matrix representing a proper rotation are $e^{\pm i\phi}$, where ϕ is the angle of rotation. 5

(q) Prove that the general displacement of a rigid body with one point fixed is a rotation about some axis. 5

B.Sc. Part-II (Semester-IV) Examination

MATHEMATICS

(Modern Algebra : Groups and Rings)

Paper—VII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives (**1 mark each**) : 10

(i) Every transposition is an :

(a) Odd permutation (b) Even permutation

(c) Both odd and even (d) None of these

(ii) If G is a finite group and H is a subgroup of G, then :

(a) 0(H) + 0(G) (b) 0(H) – 0(G)

(c) 0(G)/0(H) (d) 0(H)/0(G)

(iii) Every cyclic group is :

(a) Abelian (b) Non-abelian

(c) Cyclic (d) Infinite cyclic

(iv) The order of the identity element e of any group G is :

(a) 0 (b) 1

(c) 2 (d) 3

(v) If f be a homomorphism of a group G onto G' with Kernel K, then G' is :

(a) isomorphic to G/K (b) isomorphic to K/G

(c) isomorphic to G (d) isomorphic to K

(vi) A homomorphism of a group G into itself is :

(a) Non-homomorphism (b) Isomorphism

(c) Endomorphism (d) None of these

(vii) The intersection of two subrings is a :

(a) Division ring (b) Subring

(c) Not subring (d) None of these

(viii) A finite integral domain is a :

(a) Field (b) Prime field

(c) Sub field (d) Proper field

(ix) The intersection of two left ideals of R is :

(a) A left ideal of R (b) A right ideal of R

(c) Both left and right ideal of R (d) None of these

(x) If U is an ideal of a ring R with unity 1 and $1 \in U$ then :

(a) $U = M$ (b) $U = R$

(c) $U \neq M$ (d) $U \neq R$

UNIT—I

2. (a) Show that if every element of the group G is its own inverse, then G is abelian. 4

(b) Show that the intersection of any two subgroups of a group G is a subgroup of G. 3

(c) Show that any two distinct cycles of a permutation of a finite set are disjoint. 3

3. (p) Prove that the system (G, +) is an abelian group; with respect to '+'; where $G = \{x \mid x = a + b\sqrt{2}, a, b \in Q\}$. 4

(q) If G is a group, then prove that $(ab)^{-1} = b^{-1} a^{-1} \forall a, b \in G$. 3

(r) Prove that every cyclic group is abelian. 3

UNIT—II

4. (a) If G is a finite group and $a \in G$ then prove that $0(a) \mid 0(G)$. 4

(b) Show that every subgroup of an abelian group is normal. 3

(c) Show that if G is abelian, then the quotient group G/N is also abelian. Is its converse true ? 3

5. (p) A subgroup N of G is a normal subgroup of G iff the product of two right coset of N in G is again a right coset of N in G. Prove this. 4

(q) If N is a normal subgroup of G and H is any subgroup of G, then prove that NH is a subgroup of G. 3

(r) If $G = \{1, -1, i, -i\}$ and $N = \{1, -1\}$, then show that N is a normal subgroup of the multiplicative group G. Also find the quotient group G/N. 3

UNIT—III

6. (a) Prove that a homomorphism ϕ of G into G' with kernel K_ϕ is an isomorphism of G into G' if and only if $K_\phi = \{e\}$, where e is identity of G. 5

(b) If M, N are normal subgroups of G, then prove that :

$$\frac{NM}{M} \approx \frac{N}{N \cap M}.$$ 5

7. (p) If ϕ is a homomorphism of G into G' with kernel K, then prove that K is a normal subgroup of G. 5

(q) Let G be any group, g a fixed element in G. If $\phi : G \to G$ defined by $\phi(x) = gxg^{-1}$, then prove that ϕ is an isomorphism of G onto G. 5

UNIT—IV

8. (a) Define :

   (i) Integral domain

   (ii) Field.

   Prove that a field is an integral domain, but the converse is not true. 2+3

   (b) Prove that the characteristic of an integral domain is either zero or a prime number. 5

9. (p) Define commutative ring and prove that a ring R is commutative if and only if :

   $(a + b)^2 = a^2 + 2ab + b^2$. 1+4

   (q) Prove that a non-empty subset S of a ring R is a subring of R if and only if $x - y, xy \in S \ \forall \ x, y \in S$. 5

UNIT—V

10. (a) If R be a ring with unit elements and R not necessarily commutative such that the only right ideals of R are {0} and R, then prove that R is a division ring. 5

    (b) If U is an ideal of a ring R, then prove that R/U is a homomorphic image of R. 5

11. (p) If U and V are ideals of a ring R, then prove that :

    (i) $U \cap V$ is an ideal of R

    (ii) $U \cap V$ is the largest ideal that is contained in both U and V. 5

    (q) Let $U = \{19n \mid n \in Z\}$ be an ideal of the ring of integers Z and V be an ideal of Z with $U \subset V \subset Z$. Prove that V = U or V = Z i.e. U is a maximal ideal of Z. 5

B.Sc. Part—II (Semester—IV) Examination
MATHEMATICS
Paper—VIII
(Classical Mechanics)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve ***one*** question from each unit.

1. Choose the correct alternative :

(i) If the equation of constraint varies with time, then it is called as :

(a) Holonomic constraint

(b) Stationary or scleronomous constraint

(c) Moving or Rheonomous constraint

(d) None of these 1

(ii) The polar equation of a conic section is

$$\frac{\ell}{r} = 1 + e\cos(\theta - \theta_0)$$

where ℓ is its semi lotus rectum and e is eccentricity.

If $e < 1$, then conic represents ______.

(a) Hyperbola (b) Parabola

(c) Circle (d) Ellipse 1

(iii) If the function f(x) has maximum or minimum value at some point $x = x_0$, then the point $x = x_0$ is called as ______.

(a) Stationary point (b) Critical point

(c) Extremum point (d) None of these 1

(iv) The shortest distance between two points in a space is _____.

(a) A circle (b) A straight line

(c) An ellipse (d) A parabola 1

(v) Functions y(x) for which $\delta I[y(x)] = 0$ are called ______.

(a) Admissible functions (b) Absolute functions

(c) Stationary functions (d) None of these 1

(vi) H is the Hamiltonian of the system then a generalized coordinate q_i is said to be cyclic if _______.

(a) $\frac{\partial H}{\partial q_i} \neq 0$ (b) $\frac{\partial H}{\partial q_i} > 0$

(c) $\frac{\partial H}{\partial q_i} = 0$ (d) $\frac{\partial H}{\partial q_i} < 0$ 1

(vii) If a 3×3 matrix A is a rotation matrix, then A is orthogonal and ______.

(a) $|A| = 0$ (b) $|A| \neq 1$

(c) $|A| = 1$ (d) None of these 1

(viii) The number of degrees of freedom for a motion of a particle along a straight line are _____.

(a) 0 (b) 1

(c) 2 (d) 3 1

(ix) If H is the Hamiltonian of the system and $p_i = \frac{\partial L}{\partial \dot{q}_i}$ is the generalized momentum associated with generalized coordinate q_i, then the Hamilton's equations are ______.

(a) $\frac{\partial H}{\partial p_i} = \dot{q}_i, \ \frac{\partial H}{\partial q_i} = \dot{p}_i$ (b) $\frac{\partial H}{\partial p_i} = -\dot{q}_i, \ \frac{\partial H}{\partial q_i} = \dot{p}_i$

(c) $\frac{\partial H}{\partial p_i} = \dot{q}_i, \ \frac{\partial H}{\partial q_i} = -\dot{p}_i$ (d) $\frac{\partial H}{\partial p_i} = -\dot{q}_i, \ \frac{\partial H}{\partial q_i} = -\dot{p}_i$ 1

(x) For a particle moving under a central force such that $V = kr^{n+1}$, the virial theorem reduces to _____.

(a) $\overline{T} = (n+1)\overline{V}$ (b) $\overline{T} = (n-1)\overline{V}$

(c) $2\overline{T} = (n-1)\overline{V}$ (d) $2\overline{T} = (n+1)\overline{V}$ 1

UNIT—I

2. (a) Derive the Lagrange's equations of motion in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, i = 1, 2, \ldots n$ for conservative system from D'Alembert's principle. 6

(b) Construct a Lagrangian for a spherical pendulum and obtain the Lagrange's equations of motion. 4

OR

3. (p) Use D'Alembert's principle to obtain the equations of motion of a simple pendulum. 5

(q) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particles is $\frac{(m_1 - m_2)}{(m_1 + m_g)} g$. 5

UNIT—II

4. (a) State and prove Virial theorem. 1+4

(b) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle then the force varies as the inverse fifth power of the distance. 5

OR

5. (p) Prove that in a central force field, the areal velocity is conserved. 5

(q) Prove that if the potential energy is a homogeneous function of degree −1 in the radius vector $\bar{r}_i$, then the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 5

UNIT—III

6. (a) Find the extremals of $I[y(x)] = \int_a^b [y^2 + y'^2 + 2ye^x]dx$. 5

(b) Show that the functional :

$$I[y(x)] = \int_0^1 [2y(x) + y'(x)]dx$$

defined in the space $C_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

OR

7. (p) Prove that if x does not occur explicitly in F then, $F_{y'}y' - F$ = constant. 5

(q) Find the extremals of the functional :

$$I[y(x)] = \int_0^{\log 2} (e^{-x}y'^2 - e^x y^2)dx.$$ 5

UNIT—IV

8. (a) Obtain the Hamiltonian and then deduce the equations of motion for a simple pendulum. Show that the Hamiltonian of the system is the total energy and also the constant of motion. 6

(b) A particle moves on a smooth surface under gravity. Use Hamilton's principle to show that the equations of motion are :

$$\ddot{x} = \ddot{y} = 0, \ \ddot{z} = -g$$

where the vertical is taken along the z-axis. 4

OR

9. (p) Define : Hamiltonian H. Derive the Hamilton's equations or the canonical equations of Hamilton. 1+4

(q) Use Hamilton's principle to find the equations of motion of a particle of mass moving in space in a conservative force field F. 5

UNIT—V

10. (a) Describe the frame rotation and obtain the rotation matrix. 5

(b) If $A_1 = I + \epsilon_1$ and $A_2 = I + \epsilon_2$ be two infinitesimal rotations, then prove that infinitesimal rotations commute. 5

OR

11. (p) Prove that if A is any 2×2 orthogonal matrix with determinant $|A| = 1$, then A is a rotation matrix. 5

(q) If $A = I + \epsilon$, then prove that the inverse rotation matrix is $A^{-1} = I - \epsilon$. 5

B.Sc. (Part—II) Semester-IV Examination

MATHEMATICS

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(i) For an inverse square law, the virial theorem reduces to ______.

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = n\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -\overline{V}$

(ii) The shortest distance between two points in space is ______.

(a) A straight line (b) An ellipse

(c) A parabola (d) A circle

(iii) A bead sliding along the wire. The constraint is ______.

(a) Holonomic (b) Non-holonomic

(c) Superfluous (d) None of these

(iv) The square of the periodic time of the planet is proportional to the ______ of the major axis of its orbit.

(a) Square (b) Cube

(c) Not both (a) and (b) (d) None of these

(v) A variable quantity whose value is determined by one or more than one function is called ______.

(a) An extremum (b) A point of inflection

(c) A functional (d) None of these

(vi) The founder of the calculus of variations is ______.

(a) Lagrange (b) Leibnitz

(c) J. Bernoulli (d) Euler

(vii) If q_i is cyclic, then $\dfrac{\partial H}{\partial q_i}$ = ______.

(a) 1 (b) –1

(c) 0 (d) None of these

(viii) For a single particle system, the least action principle yield ______.

(a) $\Delta \int \sqrt{2m(H-V)}\, ds = 0$ (b) $\Delta \int \sqrt{2m(H+V)}\, ds = 0$

(c) $\Delta \int \sqrt{m(H-V)}\, ds = 0$ (d) None of these

(ix) A finite rotation can not be represented by ______.

(a) Double vector (b) Triple vector

(c) A single vector (d) None of these

(x) Infinitesimal rotation holds ______.

(a) Commutativity (b) Not Commutativity

(c) Distributivity (d) None of these

UNIT—I

2. (a) Prove virtual work on a mechanical system (for which the net virtual work of the forces of constraint vanishes) by the applied forces and the reversed effective forces is zero. 5

(b) Derive the Lagrange's equation of motion in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i'$ for a system which is partly conservative. 5

3. (p) Discuss the motion of a particle in a plane by using polar coordinates. 5

(q) If L is a Lagrangian for a system of 'n' degree of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}$, $F = F(q_1, \ldots\ldots q_n t)$ also satisfies Lagrange's equations where F is any arbitrary but differentiable function of its argument. 5

UNIT—II

4. (a) Prove for a central force field F, the path of a particle of mass m is given by

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), u = \frac{1}{r}.$$ 5

(b) Prove that for a particle moving under a central force such that $V = kr^{n+1}$, the virial theorem reduces to $2\overline{T} = (n+1)\overline{V}$. 5

5. (p) Prove the following relations :

(i) $t = \int_{r_0}^{r} \frac{dr}{f}$

(ii) $\phi = \phi_0 + \left(\frac{h}{m}\right) \int_0^t \frac{dt}{r^2}$. 3+3

(q) Prove that in a central force field, the areal velocity is conserved. 4

UNIT—III

6. (a) Find the extremals of the functional :

$$I[y(x)] = \int_0^{\log 2} (e^{-x}y'^2 - e^x y^2)\, dx. \qquad 5$$

(b) Find the shortest curve joining the points (x_1, y_1) and (x_2, y_2) in a plane. 5

7. (p) Define the n^{th} order distance. Find the second order distance between the curves $y = -\cos x$ and $y_1 = x$ on $[0, \pi/3]$. 1+4

(q) Prove that the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\, dx$ where the end points are fixed, is extremum if y satisfies the differential equation $F_y - \frac{d}{dx} Fy' = 0$. 5

UNIT—IV

8. (a) Obtain Hamilton Equations. Prove that if a generalised co-ordinate does not appear in H, then the corresponding conjugate momentum is conserved. 2+2

(b) Derive Lagrange's equations for nonholonomic conservative system. 6

9. (p) Derive the Hamilton's equations from variational principle. 5

(q) Construct the Routhian in spherical polar coordinates for a particle moving in space under the action of a conservative force field. 5

UNIT—V

10. (a) Prove that the change in the components of a vector under the infinitesimal transformation of the coordinate system is given by $d\vec{r} = \vec{r} \times d\vec{u}$. 5

(b) If A is any 2 × 2 orthogonal matrix with determinant $|A| = 1$, then prove that A is a rotation matrix. 5

11. (p) Define infinitesimal rotation. Prove that Infinitesimal rotation matrix $\in$ is antisymmetric. 5

(q) Show that the angle of rotation ϕ is given in terms of Eulerian angles by :

$$\cos\frac{\phi}{2} = \cos\frac{\theta}{2} \cdot \cos\frac{1}{2}(\phi + \psi). \qquad 5$$

B.Sc. (Part—II) Semester—IV Examination
MATHEMATICS (New)
(Classical Mechanics)
Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. 1 is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative : (1 mark each) :— 10

(i) Each planet describes ________ having the sun in one of its foci.

(a) An ellipse (b) A circle

(c) A hyperbola (d) None of these

(ii) In a central force field, the areal velocity is ________.

(a) Not constant (b) Not conserved

(c) Conserved (d) None of these

(iii) The maximum point and the minimum point of a function f(x) are called the ________.

(a) Extremum (b) Functional

(c) Continuity of a functional (d) None of these

(iv) If two curves are closed in the sense of k^{th} order proximity, then they are close in the sense of ________ order proximity.

(a) Larger (b) Smaller

(c) Equal (d) None of these

(v) Hamilton's Equation is $\dot{q}_i$ = ________.

(a) $\frac{\partial H}{\partial P_i}$ (b) $\frac{\partial H}{\partial q_i}$

(c) $\frac{\partial H}{\partial t}$ (d) None of these

(vi) If a generalised co-ordinate does not appear in H_1 then the corresponding conjugate momentum is ________ .

(a) Conserved (b) Not conserved

(c) Not constant (d) None of these

(vii) The shortest distance between two points in a plane is ________ .

(a) A straight line (b) An ellipse

(c) A parabola (d) A circle

(viii) If q_i are the generalised coordinates and the constraints are holonomic, then δq_i are ________ .

(a) Zero (b) Equivalent

(c) Dependent (d) Independent

(ix) The sum of the finite rotation depends on the ________ of the rotation.

(a) Degree (b) Order

(c) Degree and order (d) None of these

(x) The general displacement of a rigid body with ________ point fixed is a rotation about some axis.

(a) One (b) Two

(c) Three (d) None of these

UNIT—I

2. (a) Show that the shortest distance between two points in a plane is a stright line. 5

(b) Find the Lagrangian for the system consisting of a simple pendulum of mass m_2, with mass m_1 at the point of support which can move on a horizontal line lying in the plane in which m_2 moves. 5

3. (p) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common accleration of particles is $(m_1 - m_2)g/(m_1 + m_2)$. 5

(q) State and prove D'Alemberts principle. 5

UNIT—II

4. (a) Derive the differential equation for the orbit of a particle in a central force field. 5

(b) Prove that the square of the periodic time of the planet is proportional to the cube of the major axis of its orbit. 5

5. (p) Prove that in a central force field, the areal velocity is conserved. 5

(q) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5

UNIT—III

6. (a) Show that the functional :

$$I[y(x)] = \int_0^1 x^3 \sqrt{1 + y^2(x)}\, dx.$$

defined on the set of functions $y(x) \in c\,[0, 1]$ is continuous on the function $y_0(x) = x^2$ in the sense of zeroth order proximity. 5

(b) Find the extremal of the functional

$$I[y(x)] = \int_{-1}^{0} (480y - y'''^2)\, dx .$$

$$y(0) = 0,\ y(-1) = \frac{1}{3}, y'(0) = 0, y'(-1) = -2, y''(0) = 0, y''(-1) = 8.$$ 5

7. (p) Prove that if x does not occur explicitly in F, then $F_{y'}\, y' - F = \text{constant}$. 6

(q) Find the distance between the curves

$y(x) = xe^{-x}$, $y_1(x) = 0$ on $[0, 2]$. 4

UNIT—IV

8. (a) State the Hamilton's principle. Prove that Hamilton principle is a necessary and sufficient condition for Lagranges equations. 5

(b) Discuss the Routh's procedure. 5

9. (p) (i) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t} = \frac{-\partial L}{\partial t}$. 3

(ii) Prove that A cyclic co-ordinate will be absent in Hamiltonian. 3

(q) Give the physical significance of H. 4

UNIT—V

10. (a) Define Infinitisimal rotation. Prove that if $A = I + \in$, then the inverse rotation matrix $A^{-1} = I - \in$. 4

(b) Prove that the general displacement of a rigid body with one point fixed is a rotation about some axis. 6

11. (p) Define Eulerian Angle. 2

Prove that the change in the components of a vector under the infinitesimal transformation of the coordinate system is given by

$d\vec{r} = \vec{r} \times d\vec{u}$. 4

(q) Show that the two complex eigen values of an orthogonal matrix representing a proper rotation are $e^{\pm i\phi}$, where ϕ is the angle of rotation. 4

B.Sc. (Part-II) Semester-IV Examination

MATHEMATICS (NEW)

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— Question No. **1** is compulsory and attempt it once only and solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(1) The virtual work on a mechanical system by the applied forces and reversed effective forces is :

(a) Zero (b) One

(c) Negative (d) None of these

(2) If q_i is cyclic, then $\frac{\partial H}{\partial q_i} =$

(a) 0 (b) 1

(c) –1 (d) None of these

(3) A particle moving in space has ______ degrees of freedom.

(a) One (b) Two

(c) Three (d) Four

(4) A cyclic co-ordinate will be ______ in Hamiltonian.

(a) Present (b) Absent

(c) Appear (d) None of these

(5) In a central force field, the angular momentum of a particle remains :

(a) Imaginary (b) Zero

(c) Real (d) Constant

(6) For a particle moving under a central force such that $V = Kr^{n+1}$, the virial theorem reduces to :

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = (n+1)\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -(n+1)\overline{V}$

(7) A stationary point of the function f(x) includes ______.

(a) a maximum point (b) a minimum point

(c) a point of inflection (d) all of these

(8) Two curves which are close in the sense of 3rd order proximity necessarily not be close in the sense of ______ order proximity.

(a) 0^{th} (b) 1^{st}

(c) 2^{nd} (d) 4^{th}

(9) The general displacement of a rigid body with ______ point fixed is a rotation about some axis.

(a) One (b) Two

(c) Three (d) None of these

(10) ______ rotation do not commute.

(a) Infinite (b) Finite

(c) Countable (d) None of these

UNIT—I

2. (a) Derive the Lagranges equations of motion for conservative system from D'Alemberts principle. 6

(b) Find the equations of motion for a particle moving in space by using Cartesian coordinate. 4

3. (p) Construct a Lagrangian for a spherical pendulum and then obtain the Lagrange's equations of motion. 5

(q) Show that the shortest distance between two points in a plane is a straight line. 5

UNIT—II

4. (a) State and prove the virial theorem of the system. 1+4

(b) Derive the differential equation for the orbit of a particle in a central force field. 5

5. (p) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle then the force varies as the inverse fifth power of the distance. 5

(q) Derive the equation of a path of a particle in a central force field in the form :

$$\phi = \phi_0 + \left(\frac{h}{m}\right)\int_{r_0}^{r}\frac{dr}{fr^2}.$$ 5

UNIT—III

6. (a) Prove that the functional $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')\,dx$ where the end points are fixed, is extremum if y satisfies the differential equation $F_y - \frac{d}{dx} F_{y'} = 0$. 5

(b) Define N^{th} order distance between curve. Find the distance between the curves : $y(x) = x\,e^{-x}$, $y_1(x) = 0$ on [0, 2]. 1+4

7. (p) Show that the functional $I[y(x)] = \int_0^1 \{2y(x) + y'(x)\}\,dx$ defined in the space $c_1[0, 1]$ is continuous on the function $y_0(x) = x$ in the sense of first order proximity. 5

(q) Find the extremals of the functional $I[y] = \int_0^{2\pi} (y'^2 - y^2)\,dx$ that satisfies the boundary conditions $y(0) = 1$, $y(2\pi) = 1$. 5

UNIT—IV

8. (a) State and prove least action principle. 5

(b) State Hamilton's principle. Prove that :

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$ 5

9. (p) Prove that A cyclic co-ordinate will not occur in the Routhian R. 5

(q) Use Hamilton's principle to find the equations of motion of a particle of mass moving in space in a conservative force field $\vec{F}$. 5

UNIT—V

10. (a) State and prove Euler's theorem. 6

(b) Define infinitesimal rotation. Show that infinitesimal rotations commute. 4

11. (p) Prove that :

(i) If $A = I + \epsilon$, then the inverse rotation matrix $A^{-1} = I - \epsilon$. 3

(ii) Infinitesimal rotation matrix ϵ is antisymmetric. 3

(q) Prove that a rotation matrix A is orthogonal. 4

B.Sc. (Part—II) Semester—IV Examination

MATHEMATICS (NEW)

(Modern Algebra : Groups and Rings)

Paper—VII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) The identity permutation is :

(a) Even (b) Odd

(c) Even and odd (d) None of these

(ii) If N is a normal subgroup of a finite group G, then O(G/N) is equal to :

(a) $O(G) \cdot O(N)$ (b) $O(N) \mid O(G)$

(c) $O(G) \mid O(N)$ (d) None of these

(iii) The product of disjoint cycles is :

(a) Cyclic (b) Not commutative

(c) Commutative (d) None of these

(iv) Let G be a group and let $a \in G$, if $O(a) = 3$ then $O(a^{-1})$ is equal to :

(a) 0 (b) 1

(c) 2 (d) 3

(v) A homomorphism of a group into itself is :

(a) a homomorphism (b) an isomorphism

(c) an endomorphism (d) None of these

(vi) In ring R, $x^2 = x \ \forall \ x \in R$ then R is :

(a) Division ring (b) Boolean ring

(c) Ring with unity (d) Commutative ring

(vii) The ring M of 2×2 matrices is :

(a) an integral domain (b) not an integral domain

(c) commutative ring (d) None of these

(viii) An integral domain is :

(a) always a field (b) never a field

(c) a field when it is finite (d) None of these

(ix) A ring which has only trivial ideal is called :

(a) a subring (b) a proper ring

(c) a simple ring (d) None of these

(x) The intersection of two right ideals of a ring R is :

(a) a left ideal of R (b) a right ideal of R

(c) both left and right ideal of R (d) None of these 10

UNIT—I

2. (a) Prove that a group G is abelian iff $(ab)^2 = a^2b^2$, $\forall$ a, b $\in$ G. 3

(b) If s = {1, 2, 3, 4, 5} and f, g be permutations on s given by :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 4 & 2 \end{pmatrix}, \quad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

then prove that the product of permutations is not commutative. 4

(c) Prove that any cyclic group is abelian. 3

3. (p) Show that, a non-empty subset H of a group G is a subgroup of G iff :

(i) a, b $\in$ H $\Rightarrow$ ab $\in$ H,

(ii) a $\in$ H $\Rightarrow$ a^{-1} $\in$ H. 4

(q) If H_1 and H_2 are the subgroups of group G then prove that $H_1 \cap H_2$ is also a subgroup of G. 3

(r) Prove that the product of an even permutation and an odd permutation is odd. 3

UNIT—II

4. (a) Show that if G is abelian, then the quotient group G/N is also abelian. Is its converse true ? Explain. 5

(b) If H is a subgroup of G and N is a normal subgroup of G, then show that $H \cap N$ is a Normal subgroup of H. 5

5. (p) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G). 4

(q) Let H be a subgroup of G. If $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Show that N(H) is a subgroup of G. 3

(r) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N \; \forall \; g \in G$. 3

UNIT—III

6. (a) Show that any infinite cyclic group is isomorphic to the additive group of integers. 4

(b) Let G be any group and g a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that ϕ is an isomorphism of G onto G. 4

(c) Let G be a group of non-zero real numbers under multiplication and $\phi : G \to G$ such that $\phi(x) = 2^x \; \forall \; x \in G$ then prove that ϕ is not a homomorphism. 2

7. (p) If M, N are normal subgroups of G, then prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$. 5

(q) Show that the mapping $f : C \to R$ defined by $f(x + iy) = x$ is a homomorphism of the additive group of complex numbers onto the additive group of real numbers and find the Kernel of f. 5

UNIT—IV

8. (a) Prove that a ring R is commutative iff $(a + b)^2 = a^2 + 2ab + b^2$. 4

(b) Show that intersection of two subrings of a ring is a subring. 3

(c) Let the characteristic of the ring R be 2 and let $ab = ba \; \forall \; a, b \in R$. Then show that $(a + b)^2 = a^2 + b^2$. 3

9. (p) Prove that every prime field of finite characteristics $p > 0$ is isomorphic to the field z_p. 4

(q) If R is a ring with zero element 0, then for all a, b, c $\in$ R. Prove that :

(i) $a \cdot 0 = 0 \cdot a = 0$

(ii) $(-a) \cdot (-b) = a \cdot b$ 4

(r) Prove that a field is an integral domain. 2

UNIT—V

10. (a) If U and V are ideals of a ring R then prove that $U \cap V$ is the largest ideal that is contained in both U and V. 5

(b) In a principle ideal domain if p is prime and $p \mid ab$ then prove that $p \mid a$ or $p \mid b$. 5

11. (p) If U is an ideal of the ring R, then prove that R/U is a ring. 5

(q) If F is a field, then prove that its only ideals are {0} and F itself. 3

(r) Define Maximal ideal. 2

B.Sc. (Part—II) Semester-IV Examination
MATHEMATICS
Paper–VII
(Modern Algebra Groups and Rings)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative (1 mark each) : 10

(i) A group having only improper normal subgroup is called _________.

(a) a finite group (b) a permutation group

(c) a simple group (d) None of these

(ii) Every subgroup of a cyclic group is _________.

(a) non abelian (b) cyclic

(c) cyclic but not abelian (d) abelian but not cyclic

(iii) The identity permutation is _________.

(a) even (b) odd

(c) even and odd (d) even or odd

(iv) Let G be a group. Then $(ab)^{-1}$ is equal to _________.

(a) $a^{-1}b^{-1}$ (b) $b^{-1}a^{-1}$

(c) $(ba)^{-1}$ (d) None of these

(v) A homomorphism of a group into itself is _________.

(a) a homomorphism (b) an isomorphism

(c) an endomorphism (d) None of these

(vi) An integral domain has at least _________.

(a) One element (b) Two element

(c) Three element (d) None of these

(vii) If in a ring R, $x^2 = x \ \forall \ x \in R$, then R is _________.

(a) Commutative ring (b) Division ring

(c) Boolean ring (d) Ring with unity

(viii) A field which contains no proper subfield is called _________.

(a) Sub field (b) Prime field

(c) Integral domain (d) Division ring

(ix) The intersection of two left ideals of a ring R is _________.

(a) left ideal of R (b) right ideal of R

(c) both (a) and (b) (d) None of these

(x) The characteristic of an integral domain is :

(a) even number (b) odd number

(c) prime number (d) None of these .

UNIT–I

2. (a) Prove that the set $G = \{1, W, W^2\}$ is a group w.r.t. multiplication. 4

(b) Prove that every subgroup of a cyclic group is cyclic. 4

(c) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ then prove that $f.g \neq g.f$. 2

3. (p) Let G be a group. Prove that a non-empty subset H of G is a subgroup of G iff $a, b \in H \Rightarrow a.b^{-1} \in H$. 4

(q) Find whether the following permutations are even or odd : 4

(i) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$

(ii) $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 1 & 5 & 2 \end{pmatrix}$

(r) Define : 2

(i) Cyclic group

(ii) Order of an element of a group.

UNIT–II

4. (a) If H is a subgroup of a group G, then prove that any two right (left) cosets of H in G are either identical or disjoint. 5

(b) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N \ \forall \ g \in G$. 5

5. (p) Show that if G is abelian, then the quotient group G/N is also abelian. 3

(q) Let H be a subgroup of G and $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Show that H is normal in G iff $N(H) = G$. 4

(r) Prove that the intersection of two normal subgroups of a group is a normal subgroup of G. 3

UNIT–III

6. (a) If ϕ is a homomorphism of G into G′ with Kernel K, then prove that K is a normal subgroup of G. 4

(b) If ϕ is homomorphism of a group G into a group G′, then prove that :

(i) $\phi(e) = e'$ and

(ii) $\phi(x^{-1}) = (\phi(x))^{-1} \ \forall \ x \in G$

where e and e′ are identities of G and G′ respectively. 3

(c) Let G be a group of real numbers under addition and $\phi : G \to G$ such that $\phi(x) = 13x \ \forall \ x \in G$, then prove that ϕ is homomorphism. 3

7. (p) If ϕ is homomorphism of G onto G′ with Kernel K, then prove that $G/K \approx G'$. 5

(q) Define :

(i) Homomorphism

(ii) Kernel of homomorphism.

Prove that any Kernel is non-empty. 2+3

UNIT–IV

8. (a) Prove that the intersection of any family of subrings of a ring R is a sub ring of R. 3

(b) If in a ring R, $x^3 = x \ \forall \ x \in R$, then show that R is commutative. 4

(c) Let the characteristic of the ring R be 2 and let $ab = ba \ \forall \ a, b \in R$ then show that $(a + b)^2 = a^2 + b^2$. 3

9. (p) Prove that Prime field of characteristic zero is isomorphic to the field Q of rational numbers. 5

(q) Let R be a ring with a unit element, 1, in which $(ab)^2 = a^2b^2 \ \forall \ a, b \in R$. Then prove that R is commutative. 5

UNIT–V

10. (a) If U is an ideal of a ring R with unity 1 and $1 \in U$, then prove that U = R. 2

(b) If R is a commutative ring with a unit element and M is an ideal of R, then prove that M is a Maximal ideal of R iff R|M is a field. 5

(c) Let R be a commutative ring with unity. Then prove that every maximal ideal of R is a prime ideal. 3

11. (p) If U is an ideal of ring R, then prove that R|U is a homomorphic image of R. 4

(q) Let M be the ring of matrices of order 2 over the field R of real numbers and $U = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle| a, b \in R \right\}$. Prove that U is a right ideal of M but U is not left ideal. 3

(r) Let $U = \{19n \mid n \in z\}$ be an ideal of the ring of integers Z and V be an ideal of Z with $U \subset V \subset Z$. Then prove that V = U or V = Z. 3

B.Sc. (Part—II) Semester-IV Examination
MATHEMATICS
Paper–VII
(Modern Algebra Groups and Rings)

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative (1 mark each) : 10

(i) A group having only improper normal subgroup is called _________.

(a) a finite group (b) a permutation group

(c) a simple group (d) None of these

(ii) Every subgroup of a cyclic group is _________.

(a) non abelian (b) cyclic

(c) cyclic but not abelian (d) abelian but not cyclic

(iii) The identity permutation is _________.

(a) even (b) odd

(c) even and odd (d) even or odd

(iv) Let G be a group. Then $(ab)^{-1}$ is equal to _________.

(a) $a^{-1}b^{-1}$ (b) $b^{-1}a^{-1}$

(c) $(ba)^{-1}$ (d) None of these

(v) A homomorphism of a group into itself is _________.

(a) a homomorphism (b) an isomorphism

(c) an endomorphism (d) None of these

(vi) An integral domain has at least _________.

(a) One element (b) Two element

(c) Three element (d) None of these

(vii) If in a ring R, $x^2 = x \ \forall \ x \in R$, then R is _________.

(a) Commutative ring (b) Division ring

(c) Boolean ring (d) Ring with unity

(viii) A field which contains no proper subfield is called _________.

(a) Sub field (b) Prime field

(c) Integral domain (d) Division ring

(ix) The intersection of two left ideals of a ring R is _________.

(a) left ideal of R (b) right ideal of R

(c) both (a) and (b) (d) None of these

(x) The characteristic of an integral domain is :

(a) even number (b) odd number

(c) prime number (d) None of these.

UNIT–I

2. (a) Prove that the set $G = \{1, W, W^2\}$ is a group w.r.t. multiplication. 4

   (b) Prove that every subgroup of a cyclic group is cyclic. 4

   (c) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ then prove that $f.g \neq g.f$. 2

3. (p) Let G be a group. Prove that a non-empty subset H of G is a subgroup of G iff $a, b \in H \Rightarrow a.b^{-1} \in H$. 4

   (q) Find whether the following permutations are even or odd : 4

   (i) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 2 & 4 \end{pmatrix}$

   (ii) $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 1 & 5 & 2 \end{pmatrix}$

   (r) Define : 2

   (i) Cyclic group

   (ii) Order of an element of a group.

UNIT–II

4. (a) If H is a subgroup of a group G, then prove that any two right (left) cosets of H in G are either identical or disjoint. 5

   (b) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N \ \forall \ g \in G$. 5

5. (p) Show that if G is abelian, then the quotient group G/N is also abelian. 3

   (q) Let H be a subgroup of G and $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Show that H is normal in G iff $N(H) = G$. 4

   (r) Prove that the intersection of two normal subgroups of a group is a normal subgroup of G. 3

UNIT–III

6. (a) If ϕ is a homomorphism of G into G′ with Kernel K, then prove that K is a normal subgroup of G. 4

   (b) If ϕ is homomorphism of a group G into a group G′, then prove that :

   (i) $\phi(e) = e'$ and

   (ii) $\phi(x^{-1}) = (\phi(x))^{-1} \ \forall \ x \in G$

   where e and e′ are identities of G and G′ respectively. 3

   (c) Let G be a group of real numbers under addition and $\phi : G \to G$ such that $\phi(x) = 13x \ \forall \ x \in G$, then prove that ϕ is homomorphism. 3

7. (p) If ϕ is homomorphism of G onto G′ with Kernel K, then prove that $G/K \approx G'$. 5

   (q) Define :

   (i) Homomorphism

   (ii) Kernel of homomorphism.

   Prove that any Kernel is non-empty. 2+3

UNIT–IV

8. (a) Prove that the intersection of any family of subrings of a ring R is a sub ring of R. 3

(b) If in a ring R, $x^3 = x \ \forall \ x \in R$, then show that R is commutative. 4

(c) Let the characteristic of the ring R be 2 and let $ab = ba \ \forall \ a, b \in R$ then show that $(a + b)^2 = a^2 + b^2$. 3

9. (p) Prove that Prime field of characteristic zero is isomorphic to the field Q of rational numbers. 5

(q) Let R be a ring with a unit element, 1, in which $(ab)^2 = a^2b^2 \ \forall \ a, b \in R$. Then prove that R is commutative. 5

UNIT–V

10. (a) If U is an ideal of a ring R with unity 1 and $1 \in U$, then prove that U = R. 2

(b) If R is a commutative ring with a unit element and M is an ideal of R, then prove that M is a Maximal ideal of R iff R|M is a field. 5

(c) Let R be a commutative ring with unity. Then prove that every maximal ideal of R is a prime ideal. 3

11. (p) If U is an ideal of ring R, then prove that R|U is a homomorphic image of R. 4

(q) Let M be the ring of matrices of order 2 over the field R of real numbers and $U = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} | a, b \in R \right\}$. Prove that U is a right ideal of M but U is not left ideal. 3

(r) Let $U = \{19n \mid n \in z\}$ be an ideal of the ring of integers Z and V be an ideal of Z with $U \subset V \subset Z$. Then prove that V = U or V = Z. 3

B.Sc. (Part-II) Semester-IV Examination
MATHEMATICS
(Modern Algebra Groups and Rings)
Paper—VII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt at once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives (**1** mark each) : 10

(i) A nonempty subset H of the group G is a subgroup of G if and only if a, b $\in$ H $\Rightarrow$

(a) $(ab)^{-1} \in H$ (b) $ab^{-1} \in H$

(c) $a^{-1}b^{-1} \in H$ (d) None of these

(ii) The product of two even permutation is :

(a) Odd (b) Even

(c) Both odd and even (d) None of these

(iii) If G is a finite group and N is a normal subgroup of G, then O(G/N) is equal to :

(a) $O(G) \cdot O(N)$ (b) $O(G) + O(N)$

(c) $O(G) / O(N)$ (d) $O(G) - O(N)$

(iv) The subgroup N of G is a normal subgroup of G iff :

(a) $gN \neq Ng$ for some $g \in G$ (b) $gN = Ng$ for all $g \in G$

(c) $Ng = N$ for some $g \in G$ (d) $gN = N$ for all $g \in G$

(v) Let (G, +) be a group. Then mapping $\phi : G \to G$ is homomorphism if :

(a) $\phi(a + b) = \phi(a) + \phi(b)$ (b) $\phi(a \cdot b) = \phi(a) \cdot \phi(b)$

(c) $\phi(a - b) = \phi(a) - \phi(b)$ (d) $\phi\left(\frac{a}{b}\right) = \phi(a)/\phi(b)$

(vi) If ϕ be a homomorphism of group G onto G' with Kernel K, then G' is :

(a) Isomorphic to G/K (b) Isomorphic to K/G

(c) Isomorphic to G (d) One-one homomorphism

(vii) A division ring must contain at least :

(a) One element (b) Two elements

(c) Three elements (d) None of these

(viii) If in a ring R, $x^2 = x \ \forall \ x \in R$; then R is :

(a) Commutative ring (b) Division ring

(c) Boolean ring (d) Ring with unity

(ix) If U is an ideal of a ring R with unity 1 and $1 \in U$ then :

(a) $U = R$ (b) $U \neq R$

(c) $U = M$ (d) None of these

(x) A ring R has maximal ideals :

(a) If R is finite

(b) If R is finite with at least 2 elements

(c) Only if R is finite

(d) None of these

UNIT—I

2. (a) If G is an abelian group, then prove that :

$(ab)^n = a^n b^n \ \forall \ a, b \in G$ and $\forall$ integers n. 5

(b) Prove that intersection of any two subgroups of group is also a subgroup. 3

(c) If G is a group, then prove that for every $a \in G$, $(a^{-1})^{-1} = a$. 2

3. (p) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i for all $a, b \in G$, then prove that G is abelian. 4

(q) Prove that every permutation is a product of 2-cycles or transpositions. 4

(r) Prove that the identity of a group G is unique. 2

UNIT—II

4. (a) Prove that the subgroup N of G is a normal subgroup of G if and only if each left coset of N in G is a right coset of N in G. 4

(b) Let H be a subgroup of G. If $N(H) = \{g \in G / gHg^{-1} = H\}$ then prove that N(H) is a subgroup of G. 4

(c) Show that if G is abelian, then the quotient group G/N is also abelian. 2

5. (p) Let H be a subgroup of a group G. Let for $g \in G$,

$$gHg^{-1} = \{ghg^{-1} / h \in H\}$$

prove that gHg^{-1} is a subgroup of G. 4

(q) If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G. 3

(r) If H is a subgroup of G and N is a normal subgroup of G then prove that $H \cap N$ is a normal subgroup of H. 3

UNIT—III

6. (a) If ϕ is a homomorphism of a group G into a group G', then prove that :

(i) $\phi(e) = e'$

(ii) $\phi(x^{-1}) = (\phi(x))^{-1} \forall x \in G$

where e and e' are the unit elements of G and G' respectively. 4

(b) Prove that a homomorphism ϕ of G into G' with Kernel K_ϕ is an isomorphism of G into G' if and only if $K_\phi = \{e\}$, where e = identity of G. 3

(c) Let N be a normal subgroup of G. Define the mapping $\phi : G \to G/N$ such that $\phi(x) = Nx, \forall x \in G$. Then prove that ϕ is a homomorphism of G onto G/N. 3

7. (p) If ϕ be a homomorphism of G onto G' with Kernel K. Then prove that $G/K \approx G'$. 5

(q) Let ϕ be a homomorphism of G onto G' with Kernel K. Let N' be a normal subgroup of G' and $N = \{x \in G / \phi(x) \in N'\}$. Then prove that $\frac{G}{N} \approx \frac{G'}{N'}$. 5

UNIT—IV

8. (a) Prove that the set of units in a commutative ring with unity is a multiplicative abelian group. 4

(b) Let K be a nonempty subset of a field F. Then prove that K is a subfield of F if and only if $x - y, xy^{-1} \in K \; \forall \; x, y \in K, y \neq 0$. 4

(c) Define :

(i) Prime field

(ii) Ring with no zero divisor. 1+1

9. (p) Let R be a ring with a unit element 1, in which $(ab)^2 = a^2b^2 \; \forall \; a, b \in R$. Prove that R must be commutative. 5

(q) If R is a ring in which $x^2 = x \; \forall \; x \in R$, then prove that R is a commutative ring of characteristic 2. 3+2

UNIT—V

10. (a) If U is an ideal of the ring R, then prove that R/U is a ring. 4

(b) Prove that a homomorphism f of a ring R to a ring R' is an isomorphism iff Ker f = {0}. 4

(c) Define :

(i) Trivial Ideals

(ii) Simple Ring. 1+1

11. (p) If F is a field, then prove that its only ideals are {0} and F itself. 3

(q) Let R be a commutative ring and P an ideal of R. Prove that the ring of residue classes R/P is an integral domain iff P is a prime ideal. 5

(r) If U is a left ideal of a ring R, then prove that U is a subring of R. 2

B.Sc. Part—II (Semester—IV) Examination

MATHEMATICS (New)

(Modern Algebra : Groups and Rings)

Paper—VII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt at once only.

(2) Solve **one** question from each unit.

1. Choose the correct alternatives (**1** mark each) : 10

(i) The subgroup N of G is a normal subgroup of G iff :

(a) $gN \neq Ng$ for some $g \in G$ (b) $gN = Ng$ for all $g \in G$

(c) $Ng = N$ for some $g \in G$ (d) $gN = N$ for all $g \in G$

(ii) If H is a subgroup of a group G such that $H \neq \{e\}$ and $H \neq G$ then H is called :

(a) A trivial subgroup (b) Proper subgroup

(c) Improper subgroup (d) None of these

(iii) The product of two odd permutations is :

(a) Odd (b) Even

(c) Both odd and even (d) None of these

(iv) The identity element of the quotient group $G \mid H$ is :

(a) G (b) H

(c) $G \mid H$ (d) $H \mid G$

(v) A homorphism of a group into itself is :

(a) A homomorphism (b) An isomorphism

(c) An endomorphism (d) None of these

(vi) Which of the following is not an integral domain ?

(a) $(C, +, \cdot)$ (b) $(Q, +, \cdot)$

(c) $(R, +, \cdot)$ (d) $(N, +, \cdot)$

(vii) If in a ring R, $x^2 = x \ \forall x \in R$, then R is :

(a) Commutative ring (b) Division ring

(c) Boolean ring (d) Ring with unity

(viii) A field which contains no proper subfield is called :

(a) Subfield (b) Prime field

(c) Integral domain (d) Division ring

(ix) The characteristic of a finite integral domain is :

(a) Even number (b) Odd number

(c) Prime number (d) None of these

(x) A ring which has only trivial ideal is called :

(a) A subring (b) A proper ring

(c) A simple ring (d) None of these

UNIT—I

2. (a) Let G be a group then prove that $(ab)^{-1} = b^{-1} a^{-1} \ \forall \ a, b \in G$. 3

(b) Prove that every subgroup of a cyclic group is cyclic. 3

(c) Define even and odd permutation. Explain whether the following permutation is even or odd $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 2\,5\,4\,3\,6\,1\,7\,9\,8 \end{pmatrix}$. 4

3. (p) Prove that the intersection of any two subgroups of a group G is a subgroup of G. 3

(q) Prove that every permutation on a finite set is either a cycle or it can be expressed as a product of disjoint cycles. 4

(r) Let $G = \{a + b\sqrt{2} \mid a, b \in Q\}$. Show that G is a group w.r. to addition. 3

UNIT—II

4. (a) Let H be a subgroup of a group G and let a, b, $\in$ G. Then prove that Ha = Hb iff $ab^{-1} \in H$. 4

(b) Show that every subgroup of an abelian group is normal. 3

(c) If G = {1, –1, i, –i} and N = {1, –1}, then show that N is a normal subgroup of the multiplicative group G. Also find the quotient group G|N. 3

5. (p) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G). 4

(q) Prove that the subgroup N of G is a normal subgroup of G if and only if each left coset of N in G is a right coset of N in G. 4

(r) Show that if G is abelian, then quotient group G|N is also abelian. 2

UNIT—III

6. (a) Prove that any infinite cyclic group is isomorphic to the additive group of integers. 4

(b) If ϕ is an homomorphism of a group G into a group G', then prove that :

(i) $\phi(e) = e'$

(ii) $\phi(x^{-1}) = (\phi(x))^{-1} \ \forall \ x \in G$ where e and e' are the unit elements of G and G' respectively. 4

(c) Define :

(i) Endomorphism

(ii) Isomorphism. 2

7. (p) If ϕ be a homomorphism of G on to G' with Kernel K, then prove that $G \mid K \approx G'$. 5

(q) Let G is a group of nonzero real numbers under multiplication $\phi : G \rightarrow G$ such that $\phi(x) = x^2 \ \forall \ x \in G$, then prove that ϕ is homomorphism and also find its Kernel. 3+2

UNIT—IV

8. (a) If R is a ring in which $x^2 = x \ \forall \ x \in R$, then prove that R is a commutative ring of characteristic 2. 5

   (b) Let the integer $n \geq 2$ and $Z_n = \{0, 1, 2, -----, n - 1\}$. Show that Z_n is a commutative ring with unity under the addition and multiplication mod n. 5

9. (p) Prove that every prime field of characteristic zero is isomorphic to the field Q of rational numbers. 5

   (q) Prove that a finite integral domain is a field. 5

UNIT—V

10. (a) Let R be a ring $a \in R$ and $r(a) = \{x \in R \mid ax = 0\}$. Then prove that r(a) is a right ideal of R. 3

    (b) If R is a commutative ring with unity, then prove that every maximal ideal of R is a prime ideal. 3

    (c) If U is an ideal of the ring R, then prove that R/U is a ring. 4

11. (p) Let R be a ring. Then prove that the intersection of two left ideals of R is a left ideal of R. 3

    (q) Prove that a homomorphism f of a ring R to a ring R' is an isomorphism iff Ker f = {0}. 4

    (r) Prove that the ring of 2 × 2 matrices of rationals has no ideal other than {0} and the ring itself. 3