B.Sc. (Part–II) Semester–III Examination
MATHEMATICS
(Advanced Calculus)
Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(1) Every Cauchy sequence of real number is __________.

(a) unbounded (b) bounded

(c) bounded as well as unbounded (d) None of these

(2) The sequence $<s_n>$ where $s_n = \frac{n}{n+1}$ is ________.

(a) monotonically increasing (b) monotonically decreasing

(c) constant sequence (d) None of these

(3) The harmonic series $\Sigma \frac{1}{n}$ is _________.

(a) Convergent (b) Oscillatory

(c) Divergent (d) None of these

(4) Let Σa_n be a series with positive terms and $\lim_{n\to\infty} a_n^{1/n} = l$, then the series Σa_n is convergent if _______.

(a) $l = 1$ (b) $l > 1$

(c) $l < 1$ (d) None of these

(5) If $\lim_{P\to P_0} f(P) = f(P_0)$; where P, $P_0 \in R^2$ then ________.

(a) f is discontinuous at P_0 (b) f is continuous at P_0

(c) f is continuous at P (d) None of these

(6) If $\lim_{(x,y)\to(x_0,y_0)} f(x, y) = l$ exist then repeated limits are ________.

(a) equal (b) not equal

(c) not exist (d) None of these

(7) The function f(P) has absolute minima at P_0 in D if _________.

(a) $f(P) \le f(P_0)$; $\forall$ P $\in$ D (b) $f(P) \ge f(P_0)$; $\forall$ P $\in$ D

(c) $f(P) = f(P_0)$; $\forall$ P $\in$ D (d) None of these

(8) If $u = 2x - y$ and $v = x + 4y$ then $J^1 =$ ________.

(a) 1 (b) 9

(c) $\frac{1}{9}$ (d) None of these

(9) The value of $\int_1^2 \int_1^3 x^2 y \, dy \, dx$ is ________.

(a) -1 (b) $\frac{3}{28}$

(c) $\frac{28}{3}$ (d) 1

(10) The value of $\int_0^1 \int_0^1 \int_0^1 dx \, dy \, dz$ is ________.

(a) 0 (b) 2

(c) -1 (d) 1 10

UNIT—I

2. (a) Prove that a convergent sequence of a real numbers is bounded. 5

(b) Show that the sequence $<S_r>$, $S_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}$ is convergent. 5

3. (p) Prove that every convergent sequence of real numbers is a Cauchy sequence. 5

(q) Show that the sequence $<S_n>$, where $S_n = \left(1 + \frac{1}{n}\right)^n$, is convergent and that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ lies between 2 and 3. 5

UNIT—II

4. (a) Prove that the series Σx_n converges if and only if for every $\in > 0$, $\exists$ a $M(\in) \in N$ such that $m \geq n \geq M \Rightarrow | x_{n+1} + x_{n+2} + ... + x_m| < \in$. 5

(b) Test the convergence of the series $\frac{1}{x(x+2)} + \frac{1}{(x+2)(x+4)} + \frac{1}{(x+4)(x+6)} + ...$, $x \in R$, $x \neq 0$. 5

5. (p) Prove that p-series $\Sigma \frac{1}{n^p}$ is convergent for $p > 1$ and divergent for $p \leq 1$. 6

(q) Test the convergence of the series $\Sigma \frac{n^3 + a}{2^n + a} \forall n \in N$. 4

UNIT—III

6. (a) Prove that $\lim_{(x,y)\to(4,-1)} (3x-2y)=14$ by using $\in - \delta$ definition of a limit of a function. 4

(b) Expand $x^3 + y^3 - 3xy$ in powers of $x - 2$ and $y - 3$. 3

(c) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$ then prove that $f + g$ is continuous in D. 3

7. (p) Prove that the function $f(x, y) = x + y$ is continuous $\forall (x, y) \in R^2$. 4

(q) Expand e^{xy} at the point (2, 1) upto first three terms. 3

(r) Let $f(x, y) = \frac{xy}{x^2 - y^2}$, show that the simultaneous limit does not exist at the origin in spite of the fact that the repeated limits exist at the origin and each equals to zero. 3

UNIT—IV

8. (a) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. 5

(b) Find the least distance of the origin from the plane $x - 2y + 2z = 9$ by using Lagrange's method of multipliers. 5

9. (p) If x, y are differentiable functions of u, v and u, v are differentiable functions of r, s then prove that

$$\frac{\partial(x,y)}{\partial(u,v)} \frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(r,s)}.$$ 5

(q) If $xu = yz$, $yv = xz$ and $zw = xy$ find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 5

UNIT—V

10. (a) Evaluate by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy\, dx.$$ 5

(b) Evaluate $\int_v (2x+y)dv$, where v is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$. 5

11. (p) Evaluate by Stoke's theorem $\int_c (e^x dx + 2y dy - dz)$, where c is the curve $x^2 + y^2 = 4$, $z = 2$. 5

(q) Evaluate by Gauss Divergence theorem $\iint_s \bar{f}.\bar{n}\, ds$; where

$\bar{f} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ and s is the surface of rectangular parallelepiped $0 \le x \le a$; $0 \le y \le b$; $0 \le z \le c$. 5

B.Sc. (Part–II) Semester–III Examination

MATHEMATICS

(Elementary Number Theory)

Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(1) Two integers a and b that are not both zero are relatively prime whenever ________.

(a) [a, b] = 1 (b) (a, b) = 1

(c) (a, b) = d, d > 1 (d) None of these 1

(2) For $n \in N$, (n, n + 1) = ________.

(a) 1 (b) n

(c) n + 1 (d) n(n + 1) 1

(3) A linear Diophantine equation 12x + 8y = 199 has ________.

(a) unique solution (b) infinitely many solutions

(c) no solution (d) None of these 1

(4) Any two distinct Fermat numbers are ________.

(a) Composite (b) Relatively prime

(c) Prime numbers (d) None of these 1

(5) The non negative residue modulo 7 of 17 is ________.

(a) 0 (b) 1

(c) 2 (d) 3 1

(6) The inverse of 2 modulo 5 is ________.

(a) 3 (b) 2

(c) 5 (d) 1 1

(7) For any prime p, $\tau(p)$ = ________.

(a) 0 (b) 1

(c) 2 (d) None of these 1

(8) If n is divisible by a power of prime higher than one, then $\mu(n)$ = ________.

(a) 0 (b) 1

(c) n (d) n + 1 1

(9) The order of 3 modulo 5 is ________.

(a) 1 (b) 2

(c) 3 (d) 4 1

(10) A quadratic residue of 7 is ________.

(a) 3 (b) 4

(c) 5 (d) 6 1

UNIT—I

2. (a) Let $\frac{a}{b}$ and $\frac{c}{d}$ be fractions in lowest terms so that (a, b) = (c, d) = 1. Prove that if their sum is an integer, then $|b| = |d|$. 4

(b) Find the gcd of 275 and –200 and express it in the form xa + yb. 4

(c) If (a, b) = d, then show that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. 2

3. (p) Prove that a common multiple of any two non zero integers a and b is a multiple of the lcm [a, b]. 4

(q) If (a, 4) = 2 and (b, 4) = 2, then prove that (a + b, 4) = 4. 4

(r) Prove the (a, a + 2) = 1 or 2 for every integer a. 2

UNIT—II

4. (a) If P is a prime and $P \mid a_1 a_2 \ldots\ldots a_n$, then prove that P divides at least one factor a_i of the product i.e. $P \mid a_i$ for some i, where $1 \le i \le n$. 5

(b) Find the gcd and lcm of a = 18900 and b = 17160 by writing each of the numbers a and b in prime factorization canonical form. 5

5. (p) Define Fermat number. Prove that the Fermat number F_5 is divisible by 641 and hence is composite. 1+4

(q) Find the solution of the linear Diaphantine equation 5x + 3y = 52. 5

UNIT—III

6. (a) Prove that congruence modulo m is an equivalence relation. 6

(b) Solve the linear congruence

$15x \equiv 10 \pmod{25}$. 4

7. (p) Solve the system of three congruences

$x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$. 6

(q) If a, b, c and m are integers with m > 0 such that $a \equiv b \pmod m$, then prove that :

(i) $(a - c) \equiv (b - c) \pmod m$ 2

(ii) $ac \equiv bc \pmod m$. 2

UNIT—IV

8. (a) Define Euler ϕ-function. Prove that if P is a prime and k a positive integer, then

$\phi(P^k) = P^{k-1}(P - 1)$.

Evaluate $\phi(3^4)$. 1+3+1

(b) If m is a positive integer and a is an integer with (a, m) = 1, then prove that

$a^{\phi(m)} \equiv 1 \pmod{m}$. 3

(c) Prove that, for any prime P,

$\sigma(P\,!) = (P + 1)\,\sigma((P - 1)!)$. 2

9. (p) State Mobius inversion formula.

Prove that if F is a multiplicative function and $F(n) = \sum_{d/n} f(d)$, then f is also multiplicative.

1+4

(q) Let $n = p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$ be the prime factorization of the integer n > 1. If f is multiplicative function, prove that

$$\sum_{d/n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) \ldots\ldots (1 - f(p_r)).$$ 5

UNIT—V

10. (a) If P is an odd prime number, then prove that P^n has a primitive root for all positive integer n. 5

(b) Define the order of a modulo m. Given that a has order 3 modulo P, where P is an odd prime, show that a + 1 must have order 6 modulo P. 1+4

11. (p) Prove that the quadratic residues of odd prime P are congruent modulo P to the integers

$1^2, 2^2, \ldots\ldots, \left(\frac{P-1}{2}\right)^2$. 5

(q) Solve the quadratic congruence

$5x^2 - 6x + 2 \equiv 0 \pmod{13}$. 5

B.Sc. Part—II (Semester—III) Examination

MATHEMATICS (New)

Paper—V

(Advanced Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt ***one*** question from each unit.

1. Choose the correct alternative :

(i) A sequence $<S_n>$ is strictly increasing if _____ $\forall$ n $\in$ N.

(a) $S_n = S_{n+1}$ (b) $S_n \leq S_{n+1}$

(c) $S_n < S_{n+1}$ (d) $S_n > S_{n+1}$ 1

(ii) Let $\{x_n\}$ be a Cauchy sequence of real numbers. Then the sequence $\{\cos x_n\}$ is _____.

(a) Unbounded (b) Bounded but not Cauchy

(c) Cauchy but not bounded (d) Cauchy sequence 1

(iii) The P-series $\sum \frac{1}{n^P}$ is convergent for _____.

(a) P < 1 (b) P > 1

(c) P = 1 (d) P = 0 1

(iv) The series $\sum x_n = \sum \frac{1}{4^n + 1}$ is _____.

(a) Convergent (b) Divergent

(c) Harmonic (d) None of these 1

(v) If iterated limits of a function are not equal at point then :

(a) Limit exist at that point
(b) Limit does not exist
(c) Limit is zero
(d) None of these 1

(vi) If $\lim \lim_{P \to P_0} f(P) = f(P_0)$ then :

(a) f is continuous at P_0
(b) f is discontinuous at P_0
(c) f is continuous at P
(d) None of these 1

(vii) If $u = 2x - y$, $v = x + 4y$ then $J = \frac{\partial(u, v)}{\partial(x, y)} =$ ________.

(a) $\frac{1}{9}$
(b) -9
(c) 9
(d) 9^2 1

(viii) The function f(x, y) has an absolute maxima at a point (x_0, y_0) in D if ____ for all $(x, y) \in D$.

(a) $f(x, y) \leq f(x_0, y_0)$
(b) $f(x, y) \geq f(x_0, y_0)$
(c) $f_x(x, y) \leq f_x(x_0, y_0)$
(d) None of these 1

(ix) The series $\sum a r^{n-1}$ is convergent if :

(a) $r = 1$
(b) $r < 1$
(c) $r > 1$
(d) None of these 1

(x) $\int_1^2 \int_1^3 xy^2 \, dxdy =$ ______.

(a) $\frac{24}{3}$
(b) $\frac{26}{3}$
(c) $\frac{28}{3}$
(d) 10 1

UNIT—I

2. (a) Let $<x_n>$ be a sequence of real numbers that converges to $x \neq 0$. Then prove that $\lim_{n\to\infty}\left(\frac{1}{x_n}\right) = \frac{1}{x}$, for $x_n \neq 0 \ \forall\ n \in N$. 4

(b) Show that the sequence $<S_n>$ defined by $S_n = \frac{1}{3+1} + \frac{1}{3^2+1} + \ldots + \frac{1}{3^n+1}$ is monotonic and bounded. 3

(c) Every convergent sequence of real numbers is a Cauchy sequence. Prove this. 3

OR

3. (p) Show that the sequence $<S_n>$ defined by $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ does not converge. 3

(q) Let $<S_n>$ be a sequence such that $\lim S_n = \ell$ and $S_n \geq 0 \ \forall\ n \in N$. Then $\ell \geq 0$. Prove this. 3

(r) A real sequence $<S_n>$ converges if and only if for each $\in > 0$, $\exists\ M \in N$ such that $|S_m - S_n| < \in \ \forall\ m, n \geq M$. Prove this. 4

UNIT—II

4. (a) Let $\sum x_n$ be a positive term series such that $\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = \ell$.

Then the series converges if $\ell < 1$. Prove this. 4

(b) Test the convergence :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$$ 3

(c) Define :

(i) Absolutely convergent

(ii) Harmonic series

(iii) Conditionally convergent. 3

OR

5. (p) If $<a_n>$ with $a_n \geq 0$, $n \in N$ is monotonic decreasing sequence and if $\sum_{n=1}^{\infty} b_n$ is convergent then the series $\sum_{n=1}^{\infty} a_n b_n$ is also convergent. Prove this. 4

(q) Test the convergence by integral test :

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2}.$$ 3

(r) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n\to\infty} \sqrt[n]{a_n}$, $a_n \geq 0 \ \forall$ n. Then $\sum_{n=1}^{\infty} a_n$ diverges if $\ell > 1$. Prove this. 3

UNIT—III

6. (a) Let $f(x, y) = \frac{x^3}{x^2+y^2}$, for $(x, y) \neq (0, 0)$

$= 0$, for $(x, y) = (0, 0)$

Using $(\in, \delta)$ definition, prove that f is continuous at (0, 0). 3

(b) Expand $x^3 + y^3 - 3xy$ in power of $x - 2$ and $y - 3$ i.e. at the point (2, 3). 4

(c) If limit of a function f(x,y) as $(x, y) \to (x_0, y_0)$ exists, then it is unique. Prove this. 3

OR

7. (p) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$. Then prove that $f - g$ is continuous in D. 4

(q) Expand $f(x, y) = x^3 + y^3 + 3xy$ in power of $x - 1$ and $y - 1$. 3

(r) Using $\in - \delta$ definition of a limit of a function, prove that $\lim_{(x,y)\to(1,1)}(x^2+2y) = 3$. 3

UNIT—IV

8. (a) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither maxima nor minima at (0, 0). 3

(b) If $x + y + z = u$, $y + z = uv$, $z = uvw$, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. 3

(c) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 4

OR

9. (p) If x, y are differentiable functions of u, v and u, v are differentiable functions of r and s then prove that : $\frac{\partial(x,y)}{\partial(r,s)} = \frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(r,s)}$. 5

(q) Let f(x, y) be defined in an open region D and it has local maximum or local minimum at (x_0, y_0). If the partial derivatives f_x and f_y exist at (x_0, y_0), then $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$, prove this. 5

UNIT—V

10. (a) Evaluate by changing the order of integration :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x \, dy dx}{\sqrt{x^2+y^2}}.$$ 5

(b) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2) dy \, dx$. 5

OR

11. (p) Evaluate $\iint_S \overline{F}.\overline{n} \, ds$ where $\overline{F} = axi + byj + czk$ and S is the surface of sphere $x^2+y^2+z^2 = 1$. 5

(q) Evaluate by Stokes theorem $\int_C e^x dx + 2y dy - dz$, where C is the curve $x^2+y^2 = 4$, $z = 2$. 5

B.Sc. (Part—II) Semester—III Examination

MATHEMATICS

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) Every Cauchy sequence is :

(a) Unbounded (b) Bounded

(c) Oscillatory (d) None of these

(ii) The value of $\lim_{n \to \infty} \frac{4 + 3.10^n}{5 + 3.10^n}$ is :

(a) 4/5 (b) 0

(c) 4 (d) 1

(iii) If $\lim_{n \to \infty} a_n \neq 0$ then the series Σa_n is :

(a) Convergent (b) Divergent

(c) Oscillatory (d) None of these

(iv) Let Σa_n be a series of positive terms such that $\lim_{n \to \infty} \sqrt[n]{a_n} = \ell$; $\forall n$. Then Σa_n is convergent if :

(a) $\ell = 1$ (b) $\ell > 1$

(c) $\ell = 0$ (d) $\ell < 1$

(v) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) \neq f(x_0, y_0)$ then :

(a) f is continuous (b) f is continuous at (x_0, y_0)

(c) f is discontinuous (d) None of these

(vi) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) = \ell$ then the iterated limits are :

(a) Equal to ℓ (b) Greater than ℓ

(c) Less than ℓ (d) None of these

(vii) If u = 2x – y and v = x + 4y, then $\frac{\partial(u, v)}{\partial(x, y)}$ is :

(a) 7 (b) 8

(c) 1/8 (d) 9

(viii) The necessary condition for the extremum of $f(P)$ at $P_0 \in D$ is :

(a) $f_x(P_0) = 0$ (b) $f_y(P_0) = 0$

(c) $f_x(P_0) = 0$ and $f_y(P_0) = 0$ (d) $f_x(P_0) = 0$ or $f_y(P_0) = 0$

(ix) The unit normal vector $\bar{n}$ to the surface $\phi(x, y, z) = 0$ is given by :

(a) $\frac{\nabla\phi}{|\nabla\phi|}$ (b) $\nabla\phi$

(c) $\bar{k}$ (d) $\bar{j}$

(x) The value of $\int_0^2\int_0^2\int_0^2 dxdydz$ is :

(a) 6 (b) 8

(c) 4 (d) 2 10

UNIT—I

2. (a) Show that the sequence $<S_n>$ where $S_n = (1 + 1/n)^n$ is convergent and its limit lies in between 2 and 3. 5

(b) Prove that every Cauchy sequence of real numbers is bounded. 3

(c) Prove that $\lim_{n \to \infty} \frac{1 + 3 + 5 ..... + (2n - 1)}{n^2} = 1$. 2

3. (p) Prove that every monotonic sequence is convergent if and only if it is bounded. 4

(q) Prove that every convergent sequence of real numbers is a Cauchy sequence. 3

(r) Show that the sequence .2, .22, .222, .2222, ... is monotonic increasing and it will converge to 2/9. 3

UNIT—II

4. (a) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$ and diverges when $p = 1$. 4

(b) Test the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ...$. 3

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$. 3

5. (p) Let Σa_n be a series of positive terms such that $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \ell$. Then show that $\sum_{n=1}^{\infty} a_n$ is convergent if $\ell < 1$ and diverges when $\ell > 1$. 4

(q) Test the converges of the series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$. 3

(r) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n!}$. 3

UNIT—III

6. (a) Using $\in - \delta$ definition of continuity prove that $f(x, y) = x \cdot y$ is continuous for all (x, y) in xy-plane. 4

   (b) Obtain the expansion of $f(x, y) = x^2 - y^2 + 3xy$ at the point (1, 2). 3

   (c) Using $\in - \delta$ definition, prove that $\lim_{(x, y) \to (1, 2)} (x^2 + 3y) = 7$. 3

7. (p) Expand $x^3 + y^3 - 3xy$ in powers of $(x - 2)$ and $(y - 3)$. 4

   (q) If f(x, y) is continuous at $P_0(x_0, y_0)$ then prove that it is bounded in some nbd of $P_0(x_0, y_0)$. 3

   (r) Let $f(x, y) = \frac{xy}{x^2 - y^2}$. Show that simultaneous limit does not exist at the origin in spite of the fact that the repeated limits exist at the origin. 3

UNIT—IV

8. (a) Locate all critical points and determine whether a local maximum or minimum occurs at these points of $f(x, y) = x^3 - 2x^2y - x^2 - 2y^2 - 3x$. 5

   (b) Find the extreme values of $u = \frac{x}{3} + \frac{y}{4}$; subject to the condition $x^2 + y^2 = 1$. 5

9. (p) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 5

   (q) If $xu = yz$, $yv = xz$ and $zw = xy$ then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 5

UNIT—V

10. (a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$; by changing the order of integration. 5

    (b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dzdxdy$. 5

11. (p) Verify Gauss divergence theorem for the function $\bar{f} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and S is a surface of unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. 5

    (q) Verify Stoke's theorem for the function $\bar{f} = y\bar{i} + z\bar{j}$ over the plane surface $2x + 2y + z = 2$ in the first octant. 5

B.Sc. Part—II (Semester—III) Examination

MATHEMATICS (Old) (Upto S/17)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose correct alternatives :

(i) The sequence $<s_n>$, where $s_n = \frac{\sqrt{n}}{n+1}$ is :

(a) Monotonic decreasing (b) Monotonic increasing

(c) Constant sequence (d) Oscillatory sequence 1

(ii) If $<s_n>$, $<t_n>$ and $<u_n>$ be three sequences such that $s_n \le t_n \le u_n \ \forall\ n \in N$ and $\lim_{n \to \infty} s_n = \lim_{n \to \infty} u_n = \ell$ then $\lim_{n \to \infty} t_n$ is :

(a) 0 (b) ℓ

(c) $-\ell$ (d) 1 1

(iii) The geometric series $\sum_{n=1}^{\infty} x^{n-1}$ is convergent if :

(a) $x = 0$ (b) $x = 1$

(c) $x < 1$ (d) $x > 1$ 1

(iv) The series $\sum \frac{1}{n^p}$ is convergent if :

(a) $p \le 1$ (b) $p > 1$

(c) $p = 0$ (d) $p = -1$ 1

(v) If $x = r \cos \theta$, $y = r \sin \theta$ then the value of $\frac{\partial(x, y)}{\partial(r, \theta)}$ is :

(a) r (b) $-r$

(c) $r^2\theta$ (d) $r\theta$ 1

(vi) The limits $\lim_{x \to x_0}\left[\lim_{y \to y_0} F(x, y)\right]$ and $\lim_{y \to y_0}\left[\lim_{x \to x_0} F(x, y)\right]$ are called as :

(a) Left hand and right hand limits (b) Double limits

(c) Repeated or iterated limits (d) None of these 1

(vii) The value of $\beta(m, n)$ is equal to :

(a) $\Gamma(m)\Gamma(n)$ (b) $\Gamma(m)/\Gamma(n)$

(c) $\beta(n, m)$ (d) None of these 1

(viii) The improper integral $\int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0 is called :

(a) Alpha Function (b) Beta Function

(c) Gamma Function (d) None of these 1

(ix) The value of $\int_0^1\int_0^2 dx dy$ is :

(a) 1 (b) 2

(c) 3 (d) 0 1

(x) The value of $\int_0^1\int_0^1\int_0^1 dz\, dy\, dx$ is :

(a) 0 (b) 1

(c) 2 (d) 3 1

UNIT—I

2. (a) Show that the sequence $<s_n>$, $s_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)}$ is monotonic and bounded. 3

(b) Prove that if limit of sequence $<s_n>$ exists then it is unique. 4

(c) Evaluate $\lim_{n \to \infty} \frac{1 + 3 + 5 + ... + (2n-1)}{2n^2 + 1}$. 3

3. (a) If the sequence $<s_n>$ is monotone increasing and bounded above then prove that it converges to its supremum. 4

(b) Show that the sequence <.3, .33, .333, ...> is monotonic increasing and bounded above and converges to 1/3. 3

(c) Show that the sequence $<s_n>$, where $s_n = \frac{1}{n}$ is a Cauchy Sequence. 3

UNIT—II

4. (a) Using integral test, test the convergence of $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$. 3

(b) Discuss the convergence of the series $\sum \frac{n+1}{2n^2+3}$. 3

(c) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n \to \infty} \sqrt[n]{a_n}$, $a_n \geq 0 \ \forall \ n$ then prove that :

(i) $\sum_{n=1}^{\infty} a_n$ converges if $\ell < 1$ (ii) $\sum_{n=1}^{\infty} a_n$ diverges if $\ell > 1$. 4

5. (a) Test the convergence of the series $\sum \frac{n^3+a}{2^n+a}$. 3

(b) Using comparison test, test the convergence of the series :

$$1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^3} + \ldots$$ 3

(c) Prove that the geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ and diverges for $x \geq 1$. 4

UNIT—III

6. (a) Expand $x^3 + y^3 - 3xy$ in powers of $(x - 2)$ and $(y - 3)$. 3

(b) Using $\epsilon - \delta$ definition of a limit of a function, prove that $\lim_{(x, y) \to (1, 1)} (x^2 + 2y) = 3$. 3

(c) If f, g, are continuous at p_0 then prove that $f - g$ is continuous at p_0. 4

7. (a) If $x = \rho \cos \phi$, $y = \rho \sin \phi$, $z = z$. Find $\frac{\partial(x, y, z)}{\partial(\rho, \phi, z)}$. 3

(b) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 4

(c) By Lagrange's multipliers method find absolute maximum or minimum for $f(x, y) = x^2 + y^2$ where $x^4 + y^4 = 1$. 3

UNIT—IV

8. (a) Prove that $\Gamma(n + 1) = n\Gamma(n)$. 4

(b) Evaluate $\int_0^{\infty} \frac{x^3}{(1 + x)^7} dx$. 3

(c) Evaluate $\int_0^{\log 8} \int_0^{\log y} e^{x + y} dx\, dy$. 3

9. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1 + x^2}} \frac{1}{1 + x^2 + y^2} dx\, dy$. 3

(b) Prove that $\int_0^{\infty} e^{-x^{\frac{1}{n}}} dx = n\,\overline{|n}$. 3

(c) Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}$. 4

UNIT—V

10. (a) Change the order of integral $\int_0^4 \int_0^{\sqrt{4x - x^2}} f(x, y)\, dy dx$. 5

(b) Evaluate by changing the order of integration $\int_0^1 \int_x^{\sqrt{2 - x^2}} \frac{x\, dy\, dx}{\sqrt{x^2 + y^2}}$. 5

11. (a) Evaluate by changing to polar coordinates $\iint_R \frac{x^2}{\sqrt{x^2 + y^2}} dx\, dy$, where R is the region bounded by $0 \le x \le y,\ 0 \le x \le a$. 5

(b) Evaluate $\int_0^1 \int_{x^2}^{2 - x} xy\, dy\, dx$ by changing the order of integration. 5

B.Sc. (Part-II) Semester-III Examination

MATHEMATICS (New)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) The sequence $< s_n >$; where $s_n = r^n$ converges to zero if : 1

(a) $|r| < 1$ (b) $|r| > 1$

(c) $|r| = 1$ (d) None of these

(2) The value of $\lim_{n\to\infty} \frac{3^n}{2^{2n}}$ is : 1

(a) 2 (b) 1

(c) 0 (d) 4

(3) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell \ \forall_n$; then the series Σa_n is convergent if : 1

(a) $l = 1$ (b) $l < 1$

(c) $l > 1$ (d) None of these

(4) The series $1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}+.........$ is called : 1

(a) Geometric series (b) Harmonic series

(c) Arithmetic series (d) None of these

(5) The value of $\lim_{x \to 2} \left\{ \lim_{y \to 1} (xy - 3x + 4) \right\}$ is : 1

(a) 4 (b) 3

(c) 1 (d) 0

(6) The value of δ in the following expression $0 < |(x, y) - (0, 0)| < \delta \Rightarrow |x^2 + y^2| < \frac{1}{100}$ is : 1

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$

(c) 1 (d) None of these

(7) A function f(p) is said to have absolute maximum at $P_0 \in D$ iff for all $P \in D$ satisfies the condition : 1

(a) $f(P_0) \leq f(P)$ (b) $f(P_0) = f(P)$

(c) $f(P_0) \geq f(P)$ (d) None of these

(8) If $x = r \cos \theta$ and $y = r \sin \theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)}$ is : 1

(a) r (b) $\frac{1}{r}$

(c) r^2 (d) $\frac{1}{r^2}$

(9) The value of $\int_0^1 \int_0^2 \int_0^3 dxdydz$ is : 1

(a) 6. (b) 2

(c) 1 (d) 3

(10) If $\overline{F} = y\overline{i} + x\overline{j} + z^2\overline{k}$ then div $\overline{F}$ at (1, 1, 1) is : 1

(a) 2 (b) 1

(c) 0 (d) 3

UNIT—I

2. (a) If $\lim_{n\to\infty} s_n = \ell$ and $\lim_{n\to\infty} t_n = m$ then prove that :

$$\lim_{n\to\infty} s_n t_n = \ell m.$$ 4

(b) Let $< s_n >$ be a sequence such that $\lim_{n\to\infty} s_n = \ell$ and $s_n \geq 0$, then prove that $l \geq 0$. 3

(c) Prove that :

$$\lim_{n\to\infty} \frac{1+2+3+........+n}{n^2} = \frac{1}{2}.$$ 3

3. (p) Prove that limit of sequence if it exist is unique. 4

(q) Prove that the sequence $< s_n >$, $s_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$ is monotonic and bounded. 3

(r) Show that the sequence $< s_n >$ defined by $s_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ does not converge. 3

UNIT—II

4. (a) Prove that the Geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ is converges to $\frac{a}{1-r}$ if $0 < r < 1$ and diverges for $r \geq 1$. 4

(b) Test the converges of the series $\sum_{n=1}^{\infty} \frac{n}{2n^3 - 1}$. 3

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$. 3

5. (p) Let Σa_n be a series of positive terms such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \ell$. Then show that the series Σa_n is convergent if $l < 1$ and diverges when $l > 1$. 4

(q) Test the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$. 4

(r) Discuss the convergence of the series $\sum\frac{1}{4^n+1}$. 2

UNIT—III

6. (a) Prove that if limit of a function f(x, y) as $(x, y) \to (x_0, y_0)$ exist then it is unique. 4

(b) Using $\in$-δ definition, prove that :

$$\lim_{(x,y)\to(1,1)} (x^2+2y)=3.$$ 3

(c) Expand $x^3 + y^3 - 3xy$ in powers of (x – 2) and (y – 3). 3

7. (p) Using $\in$–δ definition of continuity, prove that f(x, y) = x + y is continuous for all (x, y) in xy-plane. 4

(q) Prove that $\lim_{(x,y)\to(4,-1)} (3x-2y)=14$; by using $\in - \delta$ definition. 3

(r) Expand e^{xy} at the point (2, 1) upto first three terms. 3

UNIT—IV

8. (a) A rectangular box open at the top is to have a volume of 32 cubic feet. What must be the dimensions of the box if the surface area is minimum ? 4

(b) Find the extreme values of $x^3 + y^3 - 3dxy$. 3

(c) If $u=\frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$; if $xy \neq 1$. State whether u and v are functionally related. If so, find the relationship. 3

9. (p) Find the coordinates of the foot of the perpendicular drawn from the point P(6, 2, 3) to the plane z = 5x – y + 2; by minimizing the square of the distance from P to any point (x, y, z) in the plane. 4

(q) Let the function f(x, y) be defined and continuous on an open region D of xy-plane. If f(x, y) has local maximum or minimum at $P_0(x_0, y_0)$ in D and f(x, y) is differentiable at P_0 then prove that $\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=0$ at $P_0(x_0, y_0)$. 4

(r) If $x = r \cos \theta$, $y = r \sin \theta$ then find :

$$\frac{\partial(x, y)}{\partial(r, \theta)}.$$ 2

UNIT—V

10. (a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$; by changing the order of integration. 5

(b) Evaluate $\iiint_R x^2 dx\,dy\,dz$, where R is a cube bounded by the planes $z = 0$, $z = a$, $y = 0$, $y = a$, $x = 0$, $x = a$. 5

11. (p) Verify Gauss divergence theorem for the function $\overline{F} = y\bar{i} + x\bar{j} + z^2\overline{k}$; over the region bounded by $x^2 + y^2 = 4$; $z = 0$ and $z = 2$. 5

(q) Verify Stoke's Theorem for the function $\overline{F} = x^2\bar{i} + xy\bar{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = 2$, $y = 2$. 5

B.Sc. Part—II (Semester—III) Examination
MATHEMATICS (New)
(Elementary Number Theory)
Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. 1 is compulsory; attempt it **once** only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The product of any m consecutive integers is divisible by :

(a) $(m + 1)!$ (b) $(m - 1)!$

(c) $m!$ (d) $\frac{m}{2}!$

(ii) If $c > 0$ is common divisor of a and b, then $\left(\frac{a}{c}, \frac{b}{c}\right) =$

(a) $\frac{(a, b)}{c}$ (b) $\frac{[a, b]}{c}$

(c) $\frac{c}{(a, b)}$ (d) $\frac{c}{[a, b]}$

(iii) The conjecture "Every odd integer is the sum of at most three primes" is given by :

(a) Euler (b) Goldbach

(c) Eratothenes (d) None of these

(iv) If $x > 0$, $y > 0$ and $x - y$ is even, then $x^2 - y^2$ is divisible by :

(a) 3 (b) 4

(c) 5 (d) 7

(v) If $n > 2$ is a positive integer, then :

$1^3 + 2^3 + 3^3 + 4^3 + ... + (n - 1)^3 \equiv$

(a) 0 (mod n) (b) 1 (mod n)

(c) 2 (mod n) (d) None of these

(vi) The set {0, 1, 2, 3} is complete system of residues modulo :

(a) 3 (b) 4

(c) 5 (d) 2

(vii) The function f is multiplicative, if :

(a) $f(mn) = f(m) | f(n)$ (b) $f(mn) = f(n) | f(m)$

(c) $f(mn) = f(m)\, f(n)$ (d) None of these

(viii) If n = 18, then the pair of $\tau(18)$ and $\sigma(18)$ is :

(a) (6, 39) (b) (6, 40)

(c) (7, 93) (d) (7, 92)

(ix) If $O_m(a) = n$, then $O_m(a^k) =$

(a) $\frac{m}{(m, k)}$ (b) $\frac{n}{(m, n)}$

(c) $\frac{n}{(n, k)}$ (d) None of these

(x) The quadratic residues of 7 are :

(a) (2, 3, 4) (b) (3, 5, 6)

(c) (1, 2, 4) (d) None of these 10

UNIT—I

2. (a) Find the gcd of 275 and 200 and express it in the form 275 x + 200 y. 4

(b) State and prove the division algorithm theorem. 1+3

(c) If (a, b) = d, then show that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. 2

3. (p) If $a, b \in I$, $b \neq 0$ and $a = bq + r \quad 0 \leq r < b$, then show that (a, b) = (b, r). 4

(q) For positive integer a and b, prove that :

(a, b) [a, b] = ab. 3

(r) Prove that there are no integers a, b, n > 1 such that $(a^n - b^n) \mid (a^n + b^n)$. 3

UNIT—II

4. (a) Prove that every positive integer greater than one has at least one prime divisor. 5

(b) Prove that the Fermat number F_5 is divisible by 641 and hence it is composite. 5

5. (p) Find the solution of the linear Diaphantine equation $10x + 6y = 110$. 5

(q) If P_n is the n^{th} prime number, then prove that :

$$P_n \leq 2^{2^{n-1}}.$$ 5

UNIT—III

6. (a) Show that congruence is an equivalence relation. 5

(b) Find the solution of $140x \equiv 133 \pmod{301}$. 5

7. (p) If $a \equiv b \pmod m$, then prove that $a^n \equiv b^n \pmod m$, $\forall\, n \in N$. 5

(q) Solve the system of three congruences :

$$x \equiv 1 \pmod 4,\quad x \equiv 0 \pmod 3,\quad x \equiv 5 \pmod 7.$$ 5

UNIT—IV

8. (a) If m is a positive integer and a is an integer with $(a, m) = 1$, then prove that $a^{\phi(m)} \equiv 1 \pmod m$. 5

(b) If n is a positive integer, then prove that :

$$\sum_{d|n} \phi(d) = n.$$ 5

9. (p) Prove that the Möbius r-function is multiplicative. 4

(q) For $n > 2$, prove that $\phi(n)$ is an even integer. 3

(r) Find the value of $\tau(1800)$ and $\sigma(1800)$. 3

UNIT—V

10. (a) If $O_m(a) = n$, then prove that $a^k \equiv 1 \pmod m$ iff $n|k$, $\forall\, k \in N$. 3

(b) If $(a, m) = d > 1$, then prove that m has no primitive root a. 3

(c) If p is a prime number and $d|p-1$, then prove that the congruence $x^d - 1 \equiv 0 \pmod p$ has exactly d solutions. 4

11. (p) Find all the primitive roots of $p = 17$. 5

(q) Show that the congruence $x^2 \equiv a \pmod p$ has either no solutions or exactly two incongruent solutions modulo p. 5

B.Sc. (Part—II) Semester—III Examination
MATHEMATICS
(Elementary Number Theory)
Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) :

(i) The number of multiples of 10^{44} that divides 10^{55} is :

(a) 144 (b) 11

(c) 121 (d) 12

(ii) The integers of the form $2^{2^n}+1$ are called the :

(a) Prime number (b) Ramanuj number

(c) Fermat number (d) Real number

(iii) If p is prime then $2^p + 3^p$ is :

(a) Perfect square (b) Not perfect square

(c) Positive integer (d) Negative integer

(iv) A solution of $ax \equiv 1 \pmod m$ with $(a, m) = 1$ is called an :

(a) Even number (b) Odd number

(c) Modulo m (d) Inverse of modulo m

(v) What is the total number of solutions in integers to the equation $3x + 5y = 21$?

(a) 0 (b) ∞

(c) 2 (d) 4

(vi) The set $\{0, 1, 2, 3\}$ is complete system of residues modulo :

(a) 3 (b) 4

(c) 5 (d) 2

(vii) The function f is multiplicative if :

(a) $f(m \cdot n) = f(m)|f(n)$ (b) $f(mn) = f(n)|f(m)$

(c) $f(mn) = f(m)\,f(n)$ (d) None of these

(viii) The number of primitive roots of 53 is :

(a) Zero (b) One

(c) Twenty (d) Twenty Four

(ix) The equation $m^2 - 33n + 1 = 0$, where m, n are integers, has :

(a) Exactly one solution

(b) No solution

(c) Infinitely many solutions

(d) Exactly two solutions

(x) The number of solutions to the congruence $x^3 \equiv 3 \pmod 7$ is :

(a) 3

(b) 2

(c) 1

(d) No solution 10

UNIT—I

2. (a) Let a and b be integers that both are not zero. Then prove that a and b are relatively prime iff there exist integers x and y such that $xa + yb = 1$. 4

(b) If $(a, b) = 1$, show that $(a + b, a - b) = 1$ **or** 2. 3

(c) If $a|bc$ and $(a, b) = 1$, then prove that $a|c$. 3

3. (p) State and prove Division Algorithm Theorem. 1+3

(q) If x and y are odd then prove that $x^2 + y^2$ is not a perfect square. 3

(r) Show that the square of every odd integer is of the form $8m + 1$. 3

UNIT—II

4. (a) Prove that there are infinite number of primes. 5

(b) Find the gcd and lcm of a = 18,900 and b = 17,160 by writing each of the numbers a and b in prime factorization canonical form. 5

5. (p) Prove that for all positive integer n,

$$F_0F_1 \ldots F_{n-1} = F_n - 2.$$ 5

(q) Let a and b be relatively prime integers. If d is a positive divisor of ab, then show that there is a unique pair of positive divisors d_1 of a and d_2 of b such that $d = d_1d_2$. 5

UNIT—III

6. (a) Solve the congruence :

$$7x \equiv 3 \pmod{12}.$$ 4

(b) If $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$, then prove that $a \equiv b \pmod{[m_1, m_2]}$. 4

(c) If a, b, c are integers such that $a \equiv b \pmod m$, then prove that $(a + c) \equiv (b + c) \pmod m$. 2

7. (p) If $r_1, r_2, \ldots, r_m$ is a complete system of residues modulo m and $(a, m) = 1$, a is a positive integer, then prove that $ar_1 + b, ar_2 + b, \ldots, ar_m + b$ is also complete system of residues modulo m. 5

(q) If f is a polynomial with integral coefficients and $a \equiv b \pmod m$, then prove that :

$$f(a) \equiv f(b) \pmod m.$$ 5

UNIT—IV

8. (a) Define multiplicative function. If f is a multiplicative function and $n = p_1^{a_1} \cdot p_2^{a_2} \ldots. p_m^{a_m}$ is the prime-power factorization of the positive integer n, then prove that $f(n) = f(p_1^{a_1}) \cdot f(p_2^{a_2}) \ldots. f(p_m^{a_m})$. 1+4

(b) If n is a positive integer, then prove that $\sum_{d|n} \phi(d) = n$. 5

9. (p) Prove that for each positive integer $n \geq 1$, $\sum_{d|n} \mu(d) = \begin{cases} 1, & n = 1 \\ 0, & n > 1 \end{cases}$. 5

(q) Solve the linear congruences using Euler's theorem $5x \equiv 3 \pmod{14}$. 5

UNIT—V

10. (a) Let p be an odd prime and let a be an integer with (a, p) = 1 then prove that $(a/p) \equiv a^{(p-1)/2} \bmod (p)$. 5

(b) Solve the quadratic congruence $x^2 + 7x + 10 \equiv 0 \pmod{11}$. 5

11. (p) Prove that if p is an odd prime, then $x^2 \equiv 2 \pmod{p}$ has solution iff $p \equiv \pm 1 \pmod{8}$. 5

(q) Find all primitive roots of p = 41. 5

B.Sc. Part—II (Semester—III) Examination

MATHEMATICS (New)

(Elementary Number Theory)

Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt it once only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative (1 mark each) :

(i) If c > 0 is a common multiple of a and b, then $\left(\frac{c}{a}, \frac{c}{b}\right) =$ ______ .

(a) $\frac{c}{(a, b)}$ (b) $\frac{c}{[a, b]}$

(c) $c\left(\frac{1}{a}, \frac{1}{b}\right)$ (d) None of these

(ii) For $n \geq 1$, there are at least $(n + 1)$ primes ______ 2^{2^n}

(a) Greater than (b) Less than

(c) Equal to (d) None of these

(iii) The set $\{0, 1, 2 ..., m - 1\}$ is a complete system of residues modulo :

(a) m (b) m – 1

(c) m + 1 (d) None of these

(iv) The quadratic residues of 7 are :

(a) 1, 2, 3 (b) 3, 5, 6

(c) 1, 2, 4 (d) None of these

(v) If P is a prime divisor of the Fermat number $F_n = 2^{2^n} + 1$, then $O_p(2) =$ ____.

(a) 2^n (b) 2^{n+1}

(c) 2^{2^n} (d) 2^{n-1}

(vi) The number of residues ______ the number of non residues.

(a) Equal (b) Not equal

(c) Greater than (d) Less than

(vii) If p is an odd prime, then $\left(-\frac{1}{p}\right) = -1$ if :

(a) $p \equiv 1 \pmod 4$ (b) $p \equiv -1 \pmod 4$

(c) $p \equiv 0 \pmod 4$ (d) None of these

(viii) If p is a prime, then $2^p + 3^p$ is :

(a) Perfect square (b) Not perfect square

(c) Prime (d) Positive integer

(ix) For a positive integer n, $(n-1)! \equiv -1 \pmod n \Rightarrow$ n is :

(a) Prime (b) –ve integer

(c) Positive integer (d) Composite Number

(x) If $2^3 \equiv 1 \pmod 7$; $(2, 7) = 1$, then the order of 2 modulo 7 is :

(a) 1 (b) 2

(c) 3 (d) 7 10

UNIT—I

2. (a) Find positive integers a and b satisfying the equations $(a, b) = 10$ and $[a, b] = 100$ simultaneously, find all solutions. 3

(b) Find the values of x and y to satisfy the equation $423x + 198y = 9$. 4

(c) If $(a, b) = d$, then show that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. 3

3. (p) Prove that there are no integers a, b, n > 1 such that :

$$(a^n - b^n) \mid (a^n + b^n).$$ 3

(q) If $a, b \in I$, $b \neq 0$ and $a = bq + r$, $0 \leq r < b$, then prove that $(a, b) = (b, r)$. 3

(r) Using the Euclidean algorithm find the gcd d of the number 1109 and 4999 and then find integers x and y to satisfy $d = 1109x + 4999y$. 4

UNIT—II

4. (a) If $2^m + 1$ is prime, then show that m is a power of 2, for some non negative integer K. 3

(b) Find the solution of the linear Diaphantine equation $15x + 7y = 111$. 4

(c) Show that :

$$F_0 F_1 \ldots F_{n-1} = F_{n-2}, \text{ for all positive integers.}$$ 3

5. (p) Prove that every positive integer $a > 1$ can be written uniquely as a product of primes, apart from the order in which the factors occurs i.e. $a = p_1 p_2 \ldots p_r$, all p_i being primes. 5

(q) If a prime $p > 3$, then show that $2p + 1$ and $4p + 1$ can not be prime simultaneously. 3

(r) If p is a prime and $p|_{ab}$ then show that $p|_a$ or $p|_b$. 2

UNIT—III

6. (a) If $r_1, r_2 \ldots r_m$ is a complete system of residues modulo m and $(a, m) = 1$, a is a positive integer then prove that :

$a_{r_1} + b, a_{r_2} + b \ldots a_{r_m} + b$ is also complete system of residues modulo m. 5

(b) Solve the system of three congruences :

$x \equiv 1 \pmod 4$

$x \equiv 0 \pmod 3$

$x \equiv 5 \pmod 7$. 5

7. (p) Find the solutions of $15x \equiv 12 \pmod 9$. 4

(q) Show that 41 divide $2^{20} - 1$. 3

(r) Prove that $ca \equiv cb \pmod m$ iff $a \equiv b \left(\bmod \frac{m}{d} \right)$, where $d = (c, m)$. 3

UNIT—IV

8. (a) Find the number of positive integers less or equal to 7200 that are prime to 3600. 3

(b) If $n = p_1^{a_1} p_2^{a_2} \ldots p_m^{a_m}$ is the prime-power factorization of the positive integer n, then show that :

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\ldots\left(1 - \frac{1}{p_m}\right).$$ 4

(c) If $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_m^{\alpha_m}$, then prove that :

$$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_m + 1).$$ 3

9. (p) Prove that the möbius μ-function is multiplicative. 5

(r) If m and n are two positive relatively prime integer, then show that $\phi(m\,n) = \phi(m)\,\phi(n)$. 5

UNIT—V

10. (a) If a and m are relatively prime positive integers and if a is a primitive root of m, then show that the integers $a, a^2, \ldots a^{\phi(m)}$ form a reduced residue set modulo m. 4

(b) Solve the quadratic congruence $x^2 + 7x + 10 \equiv 0 \pmod{11}$. 3

(c) If p is a prime number and $d|(p-1)$, then prove that the congruence $x^d - 1 \equiv 0 \pmod{p}$ has exactly d solutions. 3

11. (p) If p is an odd prime and a, b are integers with $(a, p) = 1 = (b, p)$ then prove that :

(i) $a \equiv b \pmod{p} \Rightarrow \left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$

(ii) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$

(iii) $\left(\frac{a^2}{p}\right) = 1.$ 5

(q) If p is a odd prime and a is a primitive root of p such that $a^{p-1} \not\equiv 1 \pmod{p^2}$, then show that for each positive integer $n \geq 2$

$$a^{p^{n-2}(p-1)} \not\equiv 1 \pmod{p^n}.$$ 5

B.Sc. (Part-II) Semester-III Examination

MATHEMATICS (New)

(Elementary Number Theory)

Paper—VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt it at once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative (**1** mark each) : 10

(1) If c > 0 is common divisor of a and b, then $\left(\frac{a}{c}, \frac{b}{c}\right)$ is equal to :

(a) $\frac{(a, b)}{c}$ (b) $\frac{[a, b]}{c}$

(c) $\frac{c}{(a, b)}$ (d) $\frac{c}{[a, b]}$

(2) The product of any m consecutive integers is divisible by :

(a) (m + 1) ! (b) (m – 1) !

(c) m ! (d) $\left(\frac{m}{2}\right)!$

(3) If x > 0, y > 0 and x – y is an even, then $(x^2 - y^2)$ is divisible by :

(a) 3 (b) 4

(c) 5 (d) 7

(4) If n > 2 is a positive integer, then $1^3 + 2^3 + \ldots\ldots + (n-1)^3 \equiv$

(a) 0 (mod n) (b) 1 (mod n)

(c) 2 (mod n) (d) None of these

(5) If (a, b) = 1 then integers a and b are :

(a) Prime (b) Relatively Primes

(c) Compositive (d) None of these

(6) An integer 'r' is root of f(x) modulo p if :

(a) $f(r) \equiv 1 \pmod p$ (b) $f(r) \equiv 0 \pmod p$

(c) $f(r) \equiv 2 \pmod p$ (d) $f(r) \equiv p \pmod 2$

(7) The number of quadratic non residues modulo 23 is :

(a) 10 (b) 22

(c) 11 (d) 2

(8) The congruence $x^n \equiv 2 \pmod{13}$ has a solution for x if :

(a) n = 5 (b) n = 7

(c) n = 6 (d) n = 8

(9) If p is a quadratic residue of an odd prime q, then q is a :

(a) quadratic residue of p (b) quadratic residue of q

(c) prime (d) residue of p

(10) By Fermat's theorem when 8^{103} is divided by 103, the remainder is :

(a) 103 (b) 8

(c) 9 (d) 10

UNIT—I

2. (a) If x and y are odd, prove that $x^2 + y^2$ is not a perfect square. 4

(b) Prove that, if $c \mid a$ and $c \mid b$, then $c \mid (a, b)$. 3

(c) Find the values of x and y to satisfy the equation $423x + 198y = 9$. 3

3. (p) If (a, b) = 1, then prove that (ac, b) = (c, b). 4

(q) For positive integers a and b, prove that :

$$(a, b)\,[a, b] = ab.$$ 3

(r) Find :

(5325, 492). 3

UNIT—II

4. (a) Prove that every positive integer greater than one has at least one prime divisor. 4

(b) Prove that :

$(a^2, b^2) = c^2$ if $(a, b) = c$. 3

(c) If P_n is the n^{th} prime number then show that :

$P_n \leq 2^{2^{n-1}}$. 3

5. (p) If m and n are distinct non-negative integers, then prove that $(F_m, F_n) = 1$. 5

(q) Find the solution of the linear Diophantine equation :

$10x + 6y = 110$. 5

UNIT—III

6. (a) Prove that congruence is an equivalence relation. 5

(b) Show that 41 divides $2^{20} - 1$. 5

7. (p) Solve the system of three congruences $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$ and $x \equiv 2 \pmod 7$. 5

(q) If f is a polynomial with integral coefficients and $a \equiv b \pmod m$, then prove that :

$f(a) \equiv f(b) \pmod m$. 5

UNIT—IV

8. (a) If p is a prime and k is a positive integer, then prove that $\phi(p^k) = p^k\left(1 - \frac{1}{p}\right)$. 5

(b) If m is a positive integer and a is an integer with $(a, m) = 1$, then prove that :

$a^{\phi(m)} \equiv 1 \pmod m$. 5

9. (p) Prove that Möbius μ-function is multiplicative. 4

(q) Find the value of $\phi(300)$. 3

(r) Find the value of $\tau(1800)$ and $\sigma(1800)$. 3

UNIT—V

10. (a) If $(a, m) = d > 1$, then prove that m has no primitive root of a. 5

(b) Prove that if r is a quadratic residue modulo $m > 2$, then $r^{\phi(m)/2} \equiv 1 \pmod m$. 5

11. (p) Let a be an odd integer, then prove that $x^2 \equiv a \pmod 4$ has a solution if and only if $a \equiv 1 \pmod 4$. 5

(q) If $m > 2$ and $n > 2$ are the integers with $(m, n) = 1$, then prove that mn has no primitive roots. 5

B.Sc. Part–II (Semester–III) Examination

MATHEMATICS (New)

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) If the limit of a sequence exists, the sequence is said to be ________.

(a) Unbounded (b) Convergent

(c) Divergent (d) Oscillatory 1

(ii) The sequence defined by $s_n = \frac{1}{n+1}$ is bounded and ______.

(a) Monotone increasing (b) Monotone decreasing

(c) Oscillatory (d) None of these 1

(iii) Let $\sum a_n$ be a series of positive terms such that $\lim_{n \to \infty} \sqrt[n]{a_n} = \ell$, $a_n \geq 0, \forall$ n. Then $\sum a_n$ is convergent if :

(a) $\ell = 1$ (b) $\ell < 1$

(c) $\ell > 1$ (d) $\ell = 0$ 1

(iv) The series $x_n = \frac{1}{n^2 + 2}$ is :

(a) Convergent (b) Divergent

(c) Oscillatory (d) None of these 1

(v) If $\lim_{(x, y) \to (x_0, y_0)} f(x, y) \neq f(x_0, y_0)$ then :

(a) f is continuous

(b) f is discontinuous

(c) function f fails to be continuous at (x_0, y_0)

(d) Both (b) and (c) 1

(vi) The neighbourhood $N_\delta(x_0, y_0) - \{(x_0, y_0)\}$ is called as :

(a) δ-nbd

(b) Rectangular nbd of (x_0, y_0)

(c) Deleted δ-nbd

(d) None of these 1

(vii) If $x = r \cos \theta$ $y = r \sin \theta$ then Jacobian $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ is :

(a) r

(b) $\frac{1}{r}$

(c) r^2

(d) $\frac{1}{r^2}$ 1

(viii) Let (x_0, y_0) be a critical point of a function of two variables which is defined in the open region $D \subseteq R^2$ and have continuous second order partial derivative in D. Then $rt - s^2 = 0 \Rightarrow$ ______.

(a) f has local maximum at (x_0, y_0)

(b) f has local minimum at (x_0, y_0)

(c) f has neither maximum nor minimum at (x_0, y_0)

(d) the test is inconclusive 1

(ix) In transforming double integral to polar co-ordinates we use dxdy =

(a) $dr d\theta$

(b) $r dr d\theta$

(c) $\frac{1}{r} dr d\theta$

(d) $\frac{dr}{d\theta}$ 1

(x) The value of $\int_0^1\int_0^1\int_0^1 dxdydz$ is :

(a) 1 (b) 0

(c) 2 (d) 3 1

UNIT—I

2. (a) Every convergent sequence of real numbers is a Cauchy Sequence. Prove this. 3

(b) Let $<s_n>$ be a sequence such that $\lim_{n\to\infty} s_n = \ell$ and $s_n \geq 0 \ \forall \ n \in N$. Then prove $\ell \geq 0$. 3

(c) Show that the sequence $<s_n>$ defined by $s_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$ converges. 4

3. (p) Prove that a monotonic sequence of real numbers is convergent if and only if it is bounded. 4

(q) Evaluate $\lim_{n\to\infty} s_n$ for $s_n = \sqrt{n+a} - \sqrt{n+b}, a \neq b$. 3

(r) Let $<x_n>$ be a sequence of real numbers and for each $n \in N$. Let $s_n = x_1 + x_2 + ... + x_n$ and $t_n = |x_1| + |x_2| + ... + |x_n|$. Prove that if $<t_n>$ is a Cauchy sequence then $<s_n>$ is Cauchy sequence. 3

UNIT—II

4. (a) Show that $\sum \frac{1}{(2n+1)^3}$ is convergent and $\sum \frac{1}{(2n-1)^{1/2}}$ is divergent. 4

(b) Let $\sum_{n=1}^{\infty} a_n$ be a sequence of real numbers such that $\ell = \lim_{n\to\infty} \sqrt[n]{a_n}, a_n \geq 0, \forall \ n$. Then prove that $\sum a_n$ is convergent if $\ell < 1$. 4

(c) A series $\sum x_n$ of non-negative terms then prove that the sequence $<s_n>$ of partial sum is monotonic increasing. 2

5. (p) Show that an absolutely convergent series is convergent but its converse necessarily does not hold. 4

(q) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, $p > 0$ by Cauchy's Integral Test. 4

(r) Test the convergence of $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots$ 2

UNIT—III

6. (a) Let f(x, y) be defined and continuous in the open region D and let $f(x_1, y_1) = z_1$, $f(x_2, y_2) = z_2$, $z_1 \neq z_2$. Then for every number z_0 between z_1 and z_2, there is a point (x_0, y_0) of D for which $f(x_0, y_0) = z_0$, prove this. 4

(b) Using $\in - \delta$ definition of a limit of a function, prove that $\lim_{(x, y) \to (4, -1)} (3x - 2y) = 14$. 3

(c) Expand $f(x, y) = x^2 - y^2 + 3xy$ at the point (1, 2) by using Taylor's theorem. 3

7. (p) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$. Then prove that f + g is continuous in D. 3

(q) Let $f(x, y) = \frac{x^2y^2}{x^2y^2 + (x - y)^2}$, $x^2y^2 + (x - y)^2 \neq 0$. Show that limit of the function f as $(x, y) \to (0, 0)$ does not exist even though iterated limits are equal. 4

(r) Expand e^{xy} at the point (2, 1) up to first three terms. 3

UNIT—IV

8. (a) If xu = yz, yv = xz, zw = xy, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 3

(b) Find the least distance of the origin from the plane x – 2y + 2z = 9 by using Lagrange's method of multipliers. 4

(c) Find the extremum of sin A sin B sin C subject to the condition A + B + C = π. 3

9. (p) Let f(x, y) be defined in an open region D and it has a local maximum or local minimum at $(x_0\ y_0)$; if the partial derivative f_x and f_y exist at (x_0, y_0), then $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. Prove this. 3

(q) If $x + y = 2e^{\theta} \cos \phi$, $x - y = 2ie^{\theta} \sin \phi$, show that $JJ' = 1$. 3

(r) Use the method of Lagrange multiplier to locate all local maxima and minima and also find the absolute maximum or minimum of $f(x, y) = x^2 + y^2$, where $x^4 + y^4 = 1$. 4

UNIT—V

10. (a) Evaluate $\iint_S \overline{F} \cdot \overline{n}\, ds$ where $\overline{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ and S is surface of rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ by Gauss-divergence theorem. 5

(b) Apply Stoke's theorem to evaluate $\oint_C [(x + y)\,dx + (2x - z)\,dy + (y + z)\,dz]$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), (0, 0, 6). 5

11. (p) Evaluate the Double integral $\int_0^{\log 8} \int_0^{\log y} e^{x+y}\, dx dy$. 3

(q) Change the order of $\iint_D f(x, y)\, dx dy$, where D is bounded by parabolas $y^2 = x$ and $x^2 = y$. 3

(r) Evaluate $\int_0^1 \int_0^{2(1-x)} \int_0^{2(1-x)-y} x^2 y\, dz dy dx$. 4