B.Sc. Part-I (Semester-II) Examination
MATHEMATICS
(Differential Equations : Ordinary & Partial)
Paper—III

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The roots of the equation $(D^2 - 4D + 13)^2 y = 0$ are : 1

(a) distinct and real (b) real and equal

(c) complex and repeated (d) None of these

(ii) A linear equation of first order is of the form Y' + PY = Q in which ? 1

(a) P is function of Y

(b) P and Q are function of X

(c) P is function of X and Q is function of Y

(d) None of these

(iii) The condition for the partial differential equation f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0 to be compatible is that : 1

(a) $J_{pp} + J_{yq} + PJ_{zp} + q.J_{zq} = 0$ (b) $J_{xp} + J_{yq} + PJ_{zp} + q.J_{zq} = 0$

(c) $J_{xp} + J_{qq} + PJ_{zp} + q.J_{zq} = 0$ (d) None of these

(iv) The D.E. $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 v}{\partial t^2}=0$ is called : 1

(a) Partial differential equation (b) Ordinary differential equation

(c) Total differential equation (d) Linear differential equation

(v) An equation of the form Pp + Qq = R where P, Q, R are the functions of X, Y, Z is called : 1

(a) Lagrange's equation (b) Jacobi's equation

(c) Charpit's equation (d) Clairaut's equation

(vi) The particular solution of DE W" + PW' + QW = 0 is $y = e^x$ iff : 1

(a) P + xQ = 0 (b) 1 + p + q = 0

(c) 1 – P + Q = 0 (d) $m^2 + mP + Q = 0$

(vii) The solution of PDE (D – mD')z = 0 is : 1

(a) z = F(y + mx) (b) z = F'(y – mx)

(c) $z = F(e^{xy})$ (d) None of these

(viii) The general form of PDE of first order is : 1

(a) $F(x, y, z, p) = 0$ (b) $F(x, y, z, q) = 0$

(c) $F(x, y, z, p, q) = 0$ (d) $F(y, z, p, q) = 0$

(ix) The complete integral of $F(x, p) = G(y, q)$ is : 1

(a) $z = \int h(x\ a)dx$ (b) $\int k(y\ a)dy$

(c) $z = \int h(x\ a)dx + \int k(y\ a)dy + b$ (d) None of these

(x) The DE $Mdx + Ndy = 0$ is exact iff : 1

(a) $\frac{\partial M}{\partial x} = \frac{\partial M}{\partial y}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

UNIT—I

2. (a) Show that the D.E. :

$$(\sin x \sin y - x e^y)dy = (e^y + \cos x \cdot \cos y)dx$$

is exact and hence solve it. 5

(b) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. 5

3. (p) Solve the D.E. :

$$(1 + x^2)dy + 2xy\ dx = \cot x\ dx.$$ 5

(q) Solve :

$$xy - \frac{dy}{dx} = y^3 e^{-x^2}.$$ 5

UNIT—II

4. (a) Solve the D.E. $(D^2 - 4)y = e^{2x}$. 5

(b) Solve the D.E. $(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$. 5

5. (p) Solve the D.E. $(x^2D^2 - xD + 4)y = \cos(\log x)$. 5

(q) Solve the D.E. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin 2x$. 5

UNIT—III

6. (a) Solve the system of D.E. : $D^2x - 2y = 0$ and $D^2y + 2x = 0$. 5

(b) Solve the D.E. $y'' - y = \frac{2}{1+e^x}$ by variation of parameter. 5

7. (p) Solve $x^2y'' + xy' + 10y = 0$ by changing the independent variable from x to $z = \log x$. 5

(q) Solve the following D.E. by removing the first derivative :

$$x\frac{d}{dx}(x\frac{dy}{dx} - y) - 2x\frac{dy}{dx} + 2y + x^2y = 0.$$ 5

UNIT—IV

8. (a) Solve :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$ 5

(b) Find the complete integral of $z = p^2x + q^2y$. 5

9. (p) Find the general solution of PDE $x^2p + y^2q = (x + y)z$. 5

(q) Solve the PDE $p^2 + q^2 = k^2$. 5

UNIT—V

10. (a) Solve the D.E. $(D^2 + 3DD' + 2D'^2)z = x + y$. 5

(b) Solve by Charpits method $pxy + pq + qy = yz$. 5

11. (p) The PDE $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ where f is homogeneous in x, y, z. Prove this. 5

(q) Find a real function v of x and y, reducing to zero when $y = 0$ and satisfying

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -4\pi(x^2 + y^2).$$ 5

B. Sc. (Part–I) Semester—II Examination

MATHEMATICS

(Vector Analysis and Solid Geometry)

Paper–IV

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory; attempt it once only.

(2) Attempt one question from each unit.

1. Choose the correct alternative :

(i) If three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, then for scalar triple product, which of the following is correct ?

(a) $\vec{b} \times \vec{c}$ is perpendicular to the vector $\vec{a}$

(b) $\vec{b} \times \vec{c}$ is parallel to the vector $\vec{a}$

(c) $\vec{b} \times \vec{c}$ is equal to the vector $\vec{a}$

(d) None of these. 1

(ii) The scalar triple product represents the volume of the _____.

(a) rectangle (b) sphere

(c) parallelepiped (d) ellipse 1

(iii) The curvature k is determined _______.

(a) only in magnitude (b) only in sign

(c) both in magnitude and sign (d) neither in magnitude nor sign 1

(iv) A plane determined by the tangent and binormal at $P(\vec{r})$ to the curve $\vec{r} = \vec{r}(s)$ is a _______.

(a) osculating plane (b) rectifying plane

(c) normal plane (d) none of these 1

(v) Which of the following quantity is defined ?

(a) div $(\text{div } \vec{f})$ (b) curl $(\text{div } \vec{f})$

(c) grad $(\text{curl } \vec{f})$ (d) grad $(\text{div } \vec{f})$ 1

(vi) A vector $\vec{f}$ is solenoidal if _______.

(a) curl $\vec{f} = 0$ (b) div $\vec{f} = 0$

(c) grad $\vec{f} = 0$ (d) grad $(\text{div } \vec{f}) = 0$ 1

(vii) If the radius of the circle is equal to the radius of the sphere, the circle is called a _____.

(a) small circle (b) imaginary circle

(c) great circle (d) none of these 1

(viii) The equations of the sphere and the plane taken together represent a _______.

(a) sphere (b) plane

(c) straight line (d) circle 1

(ix) Every section of a right circular cone by a plane perpendicular to its axis is _____.

(a) a sphere (b) a cone

(c) a circle (d) a cylinder 1

(x) The general equation of the cone passing through the coordinate axes is _______.

(a) $fyz + gzx + hxy = 0$ (b) $yz + zx + xy = 0$

(c) $ax^2 + by^2 + cz^2 = 0$ (d) $x^2 + y^2 + z^2 = 0$ 1

UNIT—I

2. (a) Show that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 5

(b) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, $\vec{b}$ and $\vec{c}$ being non-parallel. 5

3. (p) If $\vec{f}$ is a vector function of t and u is a scalar function of t, then prove that :

$$\frac{d}{dt}(u\vec{f}) = u\frac{d\vec{f}}{dt} + \frac{du}{dt}\vec{f}.$$ 5

(q) Evaluate $\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2}\, dt$, where

$$\vec{r}(t) = 5t^2\,\vec{i} + t\,\vec{j} - t^3\,\vec{k}.$$ 5

UNIT—II

4. (a) Prove that helices are the only twisted curves whose Darboux's vector has a constant direction. 5

(b) For the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at the point $t = 1$, find the equations for osculating plane, normal plane and rectifying plane. 5

5. (p) For the curve $x = a(3t - t^3)$, $y = 3at^2$, $z = a(3t + t^3)$, show that the curvature and torsion are equal. 5

(q) If $\vec{t}' = \vec{d} \times \vec{t}, \vec{n}' = \vec{d} \times \vec{n}, \vec{b}' = \vec{d} \times \vec{b}$, then find the vector $\vec{d}$. 5

UNIT—III

6. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then show that $\text{div}(r^n \vec{r}) = (n+3)r^n$. 4

(b) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point (2, – 1, 1) in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. 3

(c) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point (1, – 2, – 1). 3

7. (p) If $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$, evaluate $\int_c \vec{F}.d\vec{r}$ along the parabolic arc $y = x^2$ joining (0, 0) and (1, 1). 5

(q) Apply Green's theorem to prove that the area enclosed by a simple plane curve C is $\frac{1}{2}\int_c (xdy - ydx)$. Hence find the area of an ellipse whose semi-major and semi-minor axes are of lengths a and b. 3+2

UNIT—IV

8. (a) Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 5

(b) Find the equation of the sphere circumscribing the tetrahedron whose faces are :

$x = 0,\ y = 0,\ z = 0,\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$ 5

9. (p) State and prove the condition for the orthogonality of two spheres. 1+4

(q) Find the coordinates of the centre and radius of the circle $x + 2y + 2z = 15$; $x^2 + y^2 + z^2 - 2y - 4z = 11$. 5

UNIT—V

10. (a) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$. 5

(b) Find the equation of right circular cone whose vertical angle is 90° and its axis is along the line $x = -2y = z$. 5

11. (p) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 5

(q) Find the equation of the right circular cylinder of radius z whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, – 3, 6. 5

B.Sc. (Part-I) Semester-II Examination

2S : MATHEMATICS (New)

Differential Equation : Ordinary and Partial

Paper—III

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Solve it in one attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) The integrating factor of the DE $\frac{dy}{dx} + 2xy = x$ is ....

(a) x (b) e^x

(c) e^{x^2} (d) e^{-x^2} 1

(2) The DE Mdx + Ndy = 0 is exact if ....

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

(c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (d) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ 1

(3) The degree of the DE $\frac{d^3y}{dx^3} = \sqrt[4]{4 + \left(\frac{dy}{dx}\right)^5}$ is ....

(a) 1 (b) 2

(c) 3 (d) 4 1

(4) The primitive of the DE $\frac{d^2y}{dx^2} + 9y = 0$ is ....

(a) $y = c_1 \cos x + c_2 \sin x$ (b) $y = c_1 \cos 3x + c_2 \sin 3x$

(c) $y = (c_1 + c_2 x) \cos 3x$ (d) None of these 1

(5) The particular solution of the DE $y'' + Py' + Qy = 0$ is $y = x$ if ....

(a) $P + Qx = 0$ (b) $1 + P + Q = 0$

(c) $1 - P + Q = 0$ (d) $m^2 + mP + Q = 0$ 1

(6) The value of $\frac{1}{f(D)} e^{ax}, f(a) \neq 0$ is given by ....

(a) $\frac{1}{f(D+a)} e^{ax}$ (b) $\frac{1}{f(D-a)} e^{ax}$

(c) $\frac{1}{f(a)} e^{ax}$ (d) $\frac{1}{f(-a)} e^{ax}$ 1

(7) The general form of the First order PDE is ....

(a) $f(x, y, z, p, q) = 0$ (b) $f(x, y, p, q) = 0$

(c) $f(x, z, p, q) = 0$ (d) $f(z, p, q) = 0$ 1

(8) Lagrange's form of the PDE of order one has the form ....

(a) $P_p - Q_q = R$ (b) $P_q + Q_p = R$

(c) $P_p + Q_q = R$ (d) None of these 1

(9) The general solution of the PDE $F(D, D')Z = 0$ consists of ....

(a) C.F. (b) P.I.

(c) C.F. and P.I. (d) None of these 1

(10) The P.I. of the PDE $(D - D'^2)z = e^{2x-y}$ is ....

(a) $\frac{1}{3} e^{2x-y}$ (b) $\frac{1}{5} e^{2x-y}$

(c) $-\frac{1}{3} e^{2x-y}$ (d) e^{2x-y} 1

UNIT—I

2. (a) Solve the DE $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. 5

(b) Show that the DE $(\sin x \cdot \sin y - xe^y)\, dy = (e^y + \cos x \cdot \cos y)\, dx$ is exact and hence solve it. 5

3. (p) Solve the DE $3x^4p^2 - xp - y = 0$. 5

(q) Find the orthogonal trajectories of the family of semi cubical parabolas $ay^2 = x^3$. 5

UNIT—II

4. (a) Solve the DE $y'' - 4y' + 4y = e^{2x} + \sin 2x$. 5

(b) Solve the DE $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-5x}$. 5

5. (p) Solve the DE $(x^2D^2 - xD + 4)y = \cos(\log x)$. 5

(q) Solve the DE $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$. 5

UNIT—III

6. (a) Solve the DE $x^2y'' - 3xy' + 3y = (2x + 1)x^2$. 5

(b) Solve the DE $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ by putting $z = \sin x$. 5

7. (p) Solve the DE $y'' - y = \frac{2}{1 + e^x}$ by variation of parameters. 5

(q) Solve the simultaneous DEs $\frac{dx}{dt} + 7x - y = 0$, $\frac{dy}{dt} + 2x + 5y = 0$. 5

UNIT—IV

8. (a) Obtain the partial differential equation by eliminating arbitrary functions from :

$$V = \frac{1}{r}[f(r - at) + g(r + at)].$$ 5

(b) Solve :

$$\frac{dx}{x(y - z)} = \frac{dy}{y(z - x)} = \frac{dz}{z(x - y)}.$$ 5

9. (p) Find the general integral of the PDE $z(xp - yq) = y^2 - x^2$. 5

(q) Solve : $z^2(1 + p^2 + q^2) = k^2$. 5

UNIT—V

10. (a) Solve $(D^2 - 2DD' - 8D'^2)z = \sqrt{2x + 3y}$. 5

(b) Apply Charpit's method to solve $z^2 = pqxy$. 5

11. (p) Solve $r - 3s + 2t = e^{2x+3y} + \sin(x - 2y)$. 5

(q) Solve $D(D - 2D' - 3)z = e^{x+2y}$. 5

B.Sc. (Part—I) Semester–II Examination
MATHEMATICS (New)
Paper—III
(Differential Equations : Ordinary & Partial)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory. Solve it in **ONE** attempt only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The DE $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x is known as ______. 1

(a) Exact DE
(b) Bernoulli's equation
(c) Linear DE of order one
(d) Homogeneous DE of order one.

(ii) The order of the DE $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} - y \sin x = 0$ is ______. 1

(a) 1
(b) 2
(c) 3
(d) 4

(iii) The particular solution of the DE $y'' + Py' + Qy = 0$ is $y = e^x$ if ______. 1

(a) $P + xQ = 0$
(b) $1 + P + Q = 0$
(c) $1 - P + Q = 0$
(d) $m^2 + mP + Q = 0$

(iv) The DE $y'' - 4y' + 4y = 0$ has roots which are ______. 1

(a) real and equal
(b) real and different
(c) complex
(d) None of these

(v) The integrating factor (IF) of the DE $\frac{dy}{dx} + 2xy = x$ is ______. 1

(a) x
(b) e^x
(c) e^{x^2}
(d) e^{-x}

(vi) The value of $\frac{1}{f(D)}e^{ax}, f(a) \neq 0$ is given by ________. 1

(a) $\frac{1}{f(D+a)}e^{ax}$ (b) $\frac{1}{f(D-a)}e^{ax}$

(c) $\frac{1}{f(a)}e^{ax}$ (d) $\frac{1}{f(-a)}e^{ax}$

(vii) The correct value of $\frac{1}{f(D,D')}e^{ax+by}$ is ________. 1

(a) $\frac{1}{f(-a,-b)}e^{ax+by}$ (b) $\frac{1}{f(a,b)}e^{ax+by}$

(c) $\frac{1}{f(-a^2,-b^2)}e^{ax+by}$ (d) None of these

(viii) In PDE $P_p + Q_q = R$, where P, Q and R are functions of ________. 1

(a) x only (b) y only

(c) x and y only (d) x, y and z

(ix) Lagrange's form of the PDE of order one is ________. 1

(a) $P_p + Q_q = R$ (b) $P_p - Q_q = R$

(c) $P_q + Q_p = R$ (d) None of these

(x) The solution of the PDE $r = a^2t$ is ________. 1

(a) $z = F_1(y + ax) + F_2(y - ax)$ (b) $z = F_1(y - ax) + F_2(y - ax)$

(c) $z = F(y + ax)$ (d) None of these

UNIT—I

2. (a) Show that the DE $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$ is exact and hence solve it. 5

(b) Solve the DE $\cos x\, dy = y(\sin x - y)\, dx$. 5

3. (p) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$, where g is a parameter. 5

(q) Solve the DE $(p - xy)(p - x^2)(p - y^2) = 0$. 5

UNIT—II

4. (a) Solve the DE $\frac{d^2y}{dx^2}+a^2y = x \cos ax$. 5

(b) Solve the DE $(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$. 5

5. (p) Solve the DE $y'' + 3y' + 2y = 4x - 20 \cos 2x$. 5

(q) Solve the DE $\frac{d^2y}{dx^2}+4y = e^x + \sin 2x$. 5

UNIT—III

6. (a) Find the particular solution of $y'' - 2y' + y = 2x$ by variation of parameters. 5

(b) Solve the DE $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - y \sin^2 x = \cos x - \cos^3 x$ by changing the independent variable x to z. 5

7. (p) Solve the simultaneous DEs.

$$\frac{dx}{dt}+2\frac{dy}{dt}-2x+2y=3e^t;\ 3\frac{dx}{dt}+\frac{dy}{dt}+2x+y=4e^{2t}$$ 5

(q) Solve the DE $x^2y''-3xy'+3y=(2x+1)x^2$. 5

UNIT—IV

8. (a) Solve the PDE $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$. 5

(b) Form the PDE by eliminating the arbitrary functions from $f(x + y + z, x^2 + y^2 + z^2) = 0$. 5

9. (p) Solve the PDE $p^2 + q^2 = x^2 + y^2$. 5

(q) Solve the PDE

$$\frac{dx}{x(y^2-z^2)}=\frac{dy}{-y(z^2+x^2)}=\frac{dz}{z(x^2+y^2)}.$$ 5

UNIT—V

10. (a) Solve the PDE $r + s - 6t = y \cos x$. 5

(b) Solve the PDE $D(D - 2D' - 3)z = e^{x+2y}$ 5

11. (p) Solve the PDE $r - 3s + 2t = e^{2x+3y} + \sin(x-2y)$. 5

(q) Solve the PDE $(D^2-2DD'-8D'^2)z = \sqrt{2x+3y}.$ 5

B.Sc. (Part—I) Semester-II Examination

MATHEMATICS

(Differential Equations : Ordinary & Partial)

Paper—III

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) The order of the D.E. $\left(\frac{d^3y}{dx^3}\right)^4 - \left(\frac{dy}{dx}\right)^5 - y = 0$ is : 1

(a) 1 (b) 2

(c) 3 (d) 4

(2) The particular solution of the D.E. y'' + Py' + Qy = 0 is $y = e^x$ if : 1

(a) P + xQ = 0 (b) 1 + P + Q = 0

(c) 1 – P + Q = 0 (d) $m^2 + Pm + Q = 0$

(3) The roots of the auxiliary equations of the D.E. y'' – 5y' + 6y = 0 are : 1

(a) Real and equal (b) Complex

(c) Real and distinct (d) None of these

(4) The D.E. Mdx + Ndy = 0 is exact if : 1

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

(c) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(5) The integrating factor of the D.E. $\frac{dy}{dx} - xy = x^2$ is : 1

(a) $e^{-x^2/2}$ (b) $e^{x^2/2}$

(c) e^x (d) e^{-x}

(6) The PI of $f(D)y = e^{ax}$ is given by : 1

(a) $\frac{1}{f(D+a)} e^x$ (b) $\frac{1}{f(a)} e^x$; $f(a) \neq 0$

(c) $\frac{1}{f(D-a)} e^{ax}$ (d) $\frac{1}{f(a)} e^{ax}$; $f(a) \neq 0$

(7) Lagranges form of the PDE of order one is : 1

(a) $Pp + Qq = R$ (b) $Pp - Qq = R$

(c) $Pq + Qp = R$ (d) None of these

(8) The solution of PDE $r = a^2t$ is : 1

(a) $z = F_1(y + ax) + F_2(y - ax)$ (b) $z = F_1(y - ax) + F_2(y - ax)$

(c) $z = F(y + ax)$ (d) None of these

(9) The general solution of the PDE $F(D, D')z = 0$ is consist of : 1

(a) C.F. only (b) P.I. only

(c) C.F. and P.I. both (d) None of these

(10) The P.I. of the PDE $(2D - 3D')z = e^{x-y}$ is : 1

(a) $\frac{1}{5}e^{x-y}$ (b) $-\frac{1}{5}e^{x-y}$

(c) e^{x-y} (d) $-e^{x-y}$

UNIT—I

2. (a) Solve the D.E. $xy - \frac{dy}{dx} = y^3e^{-x^2}$. 5

(b) Show that D.E. :

$(e^y + 1) \cos x dx + e^y \sin y\, dy = 0$ is exact

and hence solve it. 5

3. (p) Find the D.E. satisfied by the system of parabolas $y^2 = 4a(x + a)$ and show that the orthogonal trajectories of the system belong to the system itself. 5

(q) Solve the D.E. $(p - xy)(p - x^2)(p - y^2) = 0$. 5

UNIT—II

4. (a) Solve the D.E. $y'' - 4y' + 4y = e^{2x} + \sin 2x$. 5

(b) Solve the D.E. $(x^2D^2 - 3xD + 5)y = x^2 \sin(\log x)$. 5

5. (p) Solve the D.E. $y'' + 3y' + 2y = e^{5x}$. 5

(q) Solve the D.E. $y'' + 2y' + 2y = x^2$. 5

UNIT—III

6. (a) Solve the D.E. $y'' - y = \frac{2}{1+e^x}$ by the method of variation of parameters. 5

(b) Solve the simultaneous DEs $\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$; $3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$. 5

7. (p) Solve the D.E. by changing the independent variable $x^6y'' + 3x^5y' + a^2y = \frac{1}{x^2}$. 5

(q) Solve the D.E. by reducing it to normal form $y'' - 2xy' + (x^2 + 2)y = e^{(x^2+2x)/2}$. 5

UNIT—IV

8. (a) Solve the PDE $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. 5

(b) Solve the PDE $p^2 + q^2 = x^2 + y^2$. 5

9. (p) Solve :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$ 5

(q) Solve the PDE $z^2(1 + p^2 + q^2) = k^2$. 5

UNIT—V

10. (a) Apply Charpit's method to solve $z^2 = pqxy$. 5

(b) Solve PDE $r - 3s + 2t = e^{2x + 3y} + \sin(x - 2y)$. 5

11. (p) Solve the PDE $D(D - 2D' - 3)z = e^{x+2y}$. 5

(q) Solve the PDE $r + s - 6t = y \cos x$. 5

B.Sc. (Part–I) Semester–II Examination

MATHEMATICS

Paper–IV

(Vector Analysis and Solid Geometry)

Time : Three Hours] [Maximum Marks : 60

N.B. :– (1) Question No. **1** is compulsory.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative :

(i) Two non-zero vectors $\bar{a}$ and $\bar{b}$ are orthogonal iff ______.

(a) $\bar{a} \cdot \bar{b} = 0$ (b) $\bar{a} \times \bar{b} = 0$

(c) $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ (d) $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ 1

(ii) The dot product of any two non-zero vectors is a ______.

(a) Vector (b) Scalar

(c) Both vector and scalar (d) None of these 1

(iii) The equation of rectifying plane is ______.

(a) $(\bar{R} - \bar{r}) \cdot \bar{b} = 0$ (b) $(\bar{R} - \bar{r}) \cdot \bar{t} = 0$

(c) $(\bar{R} - \bar{r}) \cdot \bar{n} = 0$ (d) None of these 1

(iv) A line perpendicular to both $\bar{b}$ and $\bar{n}$ is called ______.

(a) Tangent (b) Binormal

(c) Principal normal (d) None of these 1

(v) A vector $\bar{f}$ is irrotational if ______.

(a) div $\bar{f} = 0$ (b) curl $\bar{f} = 0$

(c) div grad $\bar{f} = 0$ (d) curl grad $\bar{f} = 0$ 1

(vi) If $\bar{r} = x_i + y_j + z_k$ then div $\bar{r}$ is equal to ______.

(a) Zero (b) One

(c) Two (d) Three 1

(vii) The curve of intersection of two spheres is a ______.

(a) Plane (b) Circle

(c) Sphere (d) None of these 1

(viii) The equation $x^2 + y^2 + z^2 + 4x - 6y + 10z - 11 = 0$ represents a sphere with centre $(-2, 3, -5)$ then radius of sphere is ______.

(a) 7 (b) 11

(c) 38 (d) None of these 1

(ix) Every section of a right circular cone by a plane perpendicular to its axis is a ______.

(a) Plane (b) Circle

(c) Sphere (d) Cone 1

(x) The equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represent a cone if ______.

(a) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ (b) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} < d$

(c) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} > d$ (d) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = 0$ 1

UNIT–I

2. (a) Prove that :

$$(\bar{a} \times \bar{b}) . [(\bar{b} \times \bar{c}) \times (\bar{c} \times \bar{a})] = [\bar{a}\ \bar{b}\ \bar{c}]^2$$ 4

(b) Prove that necessary and sufficient condition for $\bar{f}(t)$ to have constant magnitude is $\bar{f} . \frac{d\bar{f}}{dt} = 0.$ 3

(c) Find the value of r from the equation $\frac{d^2\bar{r}}{dt^2} = \bar{a}t + \bar{b}$, given that both $\bar{r}$ and $\frac{d\bar{r}}{dt}$ vanish when $t = 0$. 3

3. (p) If $\bar{r}$ = a cos t j + a sin t j + at tan α k, then find

$$\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right| \text{ and } \left[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}\right]$$ 4

(q) If $\overline{A} = x^2yz\,\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$ and $\overline{B} = 2z\,\vec{i} + 4\vec{j} - x^2\vec{k}$, then find $\frac{\partial^2}{\partial x \partial y}\left(\overline{A} \times \overline{B}\right)$ at (1, 0, –2) 3

(r) Prove that $\vec{i} \times (\bar{a} \times \vec{i}) + \vec{j} \times (\bar{a} \times \vec{j}) + \vec{k} \times (\bar{a} \times \vec{k}) = 2\bar{a}$. 3

UNIT–II

4. (a) For the curve $\bar{r} = \bar{r}(t)$, prove that

$$K = \frac{\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|}{|\dot{\bar{r}}|^3} \text{ and } T = \frac{\left[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}\right]}{\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|^2}.$$ 4

(b) The parametric equations of a cycloid are x = a(0 – sin 0), y = a(1 – cos 0), then show that $\rho^2 = 8ay$. 3

(c) Prove that :

$$(x''')^2 + (y''')^2 + (z''')^2 = \frac{1}{\rho^2\sigma^2} + \frac{1+\rho'^2}{\rho^4}$$ 3

5. (p) Find the curvature and torsion of the circular helix x = a cos 0, y = a sin 0, z = c0 at any point 0. 4

(q) Show that necessary and sufficient condition that a curve be a straight line is k = 0. 3

(r) If the tangent and binormal at a point of a curve make angles 0 and ϕ respectively with a fixed direction, then show that

$$\frac{\sin 0\, d0}{\sin\phi\, d\phi} = \frac{-K}{T}$$ 3

UNIT–III

6. (a) If $\overline{F} = (3x^2 + 64)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ then evaluate $\int_C \overline{F} \cdot d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$. 4

(b) Find $\nabla\phi$, if $\phi = \frac{1}{2}\log(x^2 + y^2 + z^2)$. 2

(c) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$. 4

7. (p) Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$.

where C is the boundary of the region R bounded by $y = \sqrt{x}$, $y = x^2$. 5

(q) Find the constants a, b, c so that $\vec{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. 5

UNIT–IV

8. (a) Find the equation of the sphere that passes through the circle

$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally. 5

(b) Find the equation of the sphere which passes through the points (1, –3, 4), (1, –5, 2) and (1, –3, 0) and whose centre lies on the plane $x + y + z = 0$. 5

9. (p) Prove that the two spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$ 5

(q) Find the equation of a sphere which passes through origin and intercepts lengths a, b and c on the axes respectively. 5

UNIT–V

10. (a) Find the equation of a right circular cone whose vertex is (α, β, γ), the semivertical angle α and the axis $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$. 5

(b) Find the equation of right circular cone whose vertex is (2, –3, 5), axis makes equal angles with the coordinate axes and semivertical angle is 30°. 5

11. (p) Find the equation of the right circular cylinder whose radius is r and axis the line

$$\frac{x-x'}{\ell} = \frac{y-y'}{m} = \frac{z-z'}{n}$$ 5

(q) Find the equation of right circular cylinder of radius 2 and whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}.$$ 5

B.Sc. (Part—I) Semester—II Examination
MATHEMATICS
(Vector Analysis and Solid Geometry)
Paper—IV

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose correct alternative :

(i) The cross product of any two non-zero vectors is a :

(a) Scalar (b) Vector

(c) Both Scalar and Vector (d) None of these 1

(ii) Two non-zero vectors $\bar{a}$ and $\bar{b}$ are parallel iff :

(a) $\bar{a} \cdot \bar{b} = 0$ (b) $\bar{a} \times \bar{b} = 0$

(c) $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ (d) $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ 1

(iii) The equation of osculating plane is :

(a) $(R - r) \cdot \bar{t} = 0$ (b) $(R - r) \cdot \bar{b} = 0$

(c) $(R - r) \cdot \bar{n} = 0$ (d) None of these 1

(iv) A line perpendicular to both $\bar{t}$ and $\bar{n}$ is called :

(a) tanget line (b) binormal line

(c) principal normal line (d) None of these 1

(v) A vector $\bar{f}$ is solenoidal if :

(a) $\text{div } \bar{f} = 0$ (b) $\text{curl } \bar{f} = 0$

(c) $\text{div } \bar{f} \neq 0$ (d) $\text{curl } \bar{f} \neq 0$ 1

(vi) If $\bar{r} = x_i + y_j + z_k$, then $\text{div } \bar{r}$ is equal to :

(a) Zero (b) One

(c) Two (d) Three 1

(vii) A plane section of a sphere is a :

(a) Sphere (b) Circle

(c) Both Sphere and Circle (d) None of these 1

(viii) The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a real sphere if :

(a) $u^2 + v^2 + w^2 = d$ (b) $u^2 + v^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$ (d) $u^2 + v^2 + w^2 = 0$ 1

(ix) In Right Circular Cylinder, the radius of the circle is the radius of the :

(a) Circle (b) Sphere

(c) Cylinder (d) Cone 1

(x) Every section of a right circular cone by a plane perpendicular to its axis is a :

(a) Plane (b) Circle

(c) Sphere (d) Cone 1

UNIT—I

2. (a) Prove that a necessary and sufficient condition that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$ is $(\bar{a} \times \bar{c}) \times \bar{b} = 0$. 4

(b) If f and g are functions of x, y, z then prove that $\frac{\partial}{\partial x}(\bar{f} \cdot \bar{g}) = \bar{f} \cdot \frac{\partial \bar{g}}{\partial x} + \frac{\partial \bar{f}}{\partial x} \cdot \bar{g}$. 3

(c) If $\bar{r}(t) = 5t^2\vec{i} + t\vec{j} - t^3\vec{k}$, then prove that $\int_1^2 \bar{r} \times \frac{d^2\bar{r}}{dt^2}\, dt = -14\vec{i} + 75\vec{j} - 15\vec{k}$. 3

3. (p) If $\bar{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$, $\bar{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$, $\bar{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$, then prove that $\bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{b} \cdot (\bar{c} \times \bar{a}) = \bar{c} \cdot (\bar{a} \times \bar{b})$. 4

(q) If $\bar{f} = 2t^2\vec{i} - t\vec{j} + 2\vec{k}$, $\bar{g} = 7\vec{i} + t^2\vec{j} - t\vec{k}$, then find $\frac{d}{dt}(\bar{f} \times \bar{g})$. 3

(r) Prove that :

$$(\bar{a} \times \bar{b}) \times (\bar{a} \times \bar{c}) \cdot \bar{d} = (\bar{a} \cdot \bar{d})\,[\bar{a}, \bar{b}, \bar{c}].$$ 3

UNIT—II

4. (a) Show that the Serret-Frenet formulae at a point can be written in the form $\bar{t}' = \bar{d} \times \bar{t}$, $\bar{n}' = \bar{d} \times \bar{n}$, $\bar{b}' = \bar{d} \times \bar{b}$ where $\bar{d} = \tau\bar{t} + k\bar{b}$ is a Darboux's vector. 5

(b) Prove that helices are the only twisted curves whose Darboux's vector has a constant direction. 5

5. (p) State and prove Serret-Frenet formulae. 4

(q) Find the equations of the tangent to the curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at the point $t = 1$. 3

(r) Find the curvature and torsion of the circular helix $x = a \cos \theta$, $y = a \sin \theta$, $z = c\theta$ at any point θ. 3

UNIT—III

6. (a) If $\bar{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, then evaluate $\int_C \bar{F} \cdot d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t$, $y = t^2$, $z = t^3$. 4

(b) If $\bar{r} = xi + yj + zk$ then find :

(i) grad $|\bar{r}|$

(ii) div. $\bar{r}$

(iii) curl $\bar{r}$. 2+2+2

7. (p) Verify Green's theorem in the plane for $\int_C (xy + y^2)\, dx + x^2\, dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 4

(q) If $\bar{f} = x^2 z\vec{i} - 2y^3 z^2 \vec{j} + xy^2 z\vec{k}$, then find div $\bar{f}$ and curl $\bar{f}$ at (1, −1, 1). 3

(r) Find the work done in moving a particle once around a circle C in the xy plane of radius 2 and centre (0, 0) and if the force field is given by $f = 3xy\vec{i} - y\vec{j} + 2zx\vec{k}$. 3

UNIT—IV

8. (a) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 5

(b) Find the equation to the sphere which passes through the points (0, 0, 0), (0, 1, −1), (−1, 2, 0) and (1, 2, 3). 5

9. (p) Show that the spheres :

$x^2 + y^2 + z^2 + 2x - 6y - 14z + 1 = 0$ and

$x^2 + y^2 + z^2 - 4x - 8y + 2z + 5 = 0$ are orthogonal. 5

(q) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). 5

UNIT—V

10. (a) Find the equation of right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 5

(b) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$. 5

11. (p) Prove that the equation of a cone with vertex at the origin is homogeneous. 5

(q) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$. 5

B.Sc. (Part—I) Semester—II Examination
MATHEMATICS
Paper—IV
(Vector Analysis & Solid Geometry)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose correct alternative of the following :—

(i) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplaner iff _______.

(a) $\bar{a} \times (\bar{b} \times \bar{c}) = \bar{0}$ (b) $\bar{a} \cdot (\bar{b} \times \bar{c}) = 0$

(c) $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{0}$ (d) $(\bar{a} + \bar{b}) \times \bar{c} = \bar{0}$ 1

(ii) A vector $\bar{f}$ is irrotational if _______.

(a) div $\bar{f} = 0$ (b) div $\bar{f} \neq 0$

(c) curl $\bar{f} = \bar{0}$ (d) None of these 1

(iii) If $\bar{r} = t\bar{i} + \sin t\,\bar{j} + (t^2 - 1)\vec{k}$, then $\dot{\bar{r}}$ at t = 0 is _______.

(a) (0, 0, 1) (b) (0, 1, 0)

(c) (1, 1, 0) (d) (1, 0, 1) 1

(iv) For any space curve, $\bar{t}' \cdot \bar{b}' =$ _______.

(a) k (b) J

(c) kJ (d) –kJ 1

(v) If $\bar{r} = \bar{r}(t)$ is equation of space curve, then the curvature k is equal to _______.

(a) $\dfrac{[\dot{\bar{r}}\ \ddot{\bar{r}}\ \dddot{\bar{r}}]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}$ (b) $\dfrac{\dot{\bar{r}}}{|\dot{\bar{r}}|}$

(c) $\dfrac{\dot{\bar{r}} \times \ddot{\bar{r}}}{|\dot{\bar{r}} \times \ddot{\bar{r}}|}$ (d) $\dfrac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$ 1

(vi) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then div. $\bar{r}$ is ______.

(a) 3 (b) -2

(c) 0 (d) -1 1

(vii) A vector $\bar{f}$ is solenoidal if ______.

(a) div. $\bar{f} = 0$ (b) curl $\bar{f} = \bar{0}$

(c) div. grad $\bar{f} = 0$ (d) curl grad $\bar{f} = \bar{0}$ 1

(viii) Every section of right circular cone by a plane perpendicular to its axis is ______.

(a) plane (b) circle

(c) sphere (d) None of these 1

(ix) The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represent a real sphere if ______.

(a) $u^2 + v^2 + w^2 = d$ (b) $u^2 + v^2 + w^2 > d$

(c) $u^2 + v^2 + w^2 < d$ (d) $u^2 + v^2 + w^2 = 0$ 1

(x) Two non-parallel planes intersect in a ______.

(a) plane (b) point

(c) line (d) circle 1

UNIT—I

2. (a) If vectors $\bar{f}$ and $\bar{g}$ are vector functions of t, then prove that

$$\frac{d}{dt}(\bar{f} \circ \bar{g}) = \bar{f} \circ \frac{d\bar{g}}{dt} + \frac{d\bar{f}}{dt} \circ \bar{g}.$$ 3

(b) Prove that $\bar{r} = \bar{a}\,e^{mt} + \bar{b}\,e^{nt}$, where $\bar{a}$, $\bar{b}$ are unit vectors is the solution of

$$\frac{d^2\bar{r}}{dt^2} - (m+n)\frac{d\bar{r}}{dt} + mn\,\bar{r} = 0.$$ 3

(c) If $\bar{f} = 2t^2\vec{i} - t\vec{j} + 2\vec{k}$ and $\bar{g} = 7\vec{i} + t^2\vec{j} - t\vec{k}$, then find $\frac{d}{dt}(\bar{f} \times \bar{g})$. 4

3. (p) Prove that :

$$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}.$$ 4

(q) If $\bar{a} = t\vec{i} - 3\vec{j} + 2t\vec{k}$, $\bar{b} = \vec{i} - 2\vec{j} + 2\vec{k}$ and $\bar{c} = 3\vec{i} + t\vec{j} - \vec{k}$, then evaluate $\bar{a}.(\bar{b}\times\bar{c})$. 3

(r) Prove that :

$$(\bar{c}\times\bar{a})\times(\bar{a}\times\bar{b}) = [\bar{a}\ \bar{b}\ \bar{c}]\,\bar{a}.$$ 3

UNIT—II

4. (a) State and prove Frenet-Serret formulae. 1+5

(b) If tangent and binormal at a point of a curve makes angle θ, φ respectively with fixed direction, then show that :

$$\frac{\sin\theta\, d\theta}{\sin\phi\, d\phi} = \frac{-k}{J}$$ 4

5. (p) Prove that $[\bar{r}'',\ \bar{r}''',\ \bar{r}''''] = k^3[kJ' - k'J]$. 3

(q) Show that the necessary and sufficient condition that a curve to be a straight line is k = 0. 3

(r) Prove that Darboux vector $\bar{d}$ has fixed direction if and only if k/J is constant. 4

UNIT—III

6. (a) Find the work done in moving a particle along the parabola $y^2 = x$ in the xy plane from (0, 0) to (1, 1) if the force field is given by :

$$\bar{f} = (2x + y - 7z)\bar{i} + (7x - 2y + 2z^2)\bar{j} + (3x - 2y + 4z^3)\bar{k}.$$ 5

(b) Verify Green's theorem in the plane for,

$$\int_c (xy + y^2)dx + x^2 dy.$$

Where c is the closed curve of the region bounded by y = x and $y = x^2$. 5

7. (p) If $\bar{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then evaluate $\int_c \bar{F}.d\bar{r}$ from (0, 0, 0) to (1, 1, 1) along the path x = t, $y = t^2$, $z = t^3$. 5

(q) Prove that $r^n\,\bar{r}$ is irrotational. Find the value of n when it is solenoidal. 5

UNIT—IV

8. (a) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 5

(b) Prove that the two spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$

and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$

will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$. 5

9. (p) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally. 5

(q) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 5

UNIT—V

10. (a) Find the equation of the right circular cylinder of radius 2 and whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}.$$ 5

(b) Find the equation of the right circular cylinder whose radius is r and axis the line :

$$\frac{x-x'}{l} = \frac{y-y'}{m} = \frac{z-z'}{n}.$$ 5

11. (p) Find the equation of a right circular cone whose vertex is (α, β, γ), the semivertical angle α and the axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$. 5

(q) Find the equation of right circular cone whose vertex is (2, –3, 5), axis makes equal angles with the coordinate axes and semi vertical angle is 30°. 5

B.Sc. (Part-I) Semester-II Examination

2S : MATHEMATICS

Vector Analysis & Solid Geometry

Paper—IV

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternatives of the following :

(1) Volume of parallelepiped with $\bar{a}, \bar{b}, \bar{c}$ as edge vectors is : 1

(a) $\bar{a} \times (\bar{b} \times \bar{c})$ (b) $\bar{a} \cdot (\bar{b} \times \bar{c})$

(c) $(\bar{a} \times \bar{b}) \times \bar{c}$ (d) $(\bar{a} + \bar{b}) \times \bar{c}$

(2) Scalar triple product containing two repeated vectors is : 1

(a) Less than zero (b) Equal to zero

(c) Not equal to zero (d) Greater than zero

(3) The curve of intersection of two spheres is : 1

(a) Circle (b) Point

(c) Line (d) Plane

(4) Every homogeneous equation of second degree in x, y and z, represent a ______ whose vertex is at the origin. 1

(a) Cone (b) Cylinder

(c) Sphere (d) None of these

(5) A helix is a twisted curve whose tangent makes a constant angle with a : 1

(a) Tangent (b) Plane

(c) Fixed direction (d) Binormal

(6) The plane which passes through $P(\bar{\varepsilon})$ and contains binormal and tangent is said to be : 1

(a) Osculating plane (b) Rectifying plane

(c) Normal plane (d) None of these

(7) If $\bar{\varepsilon} = \bar{\varepsilon}(t)$ is equation of space curve, then curvature is equal to : 1

(a) $\dfrac{[\dot{\varepsilon}\ \ddot{\varepsilon}\ \dddot{\varepsilon}]}{|\dot{\varepsilon} \times \ddot{\varepsilon}|^2}$

(b) $\dfrac{\dot{\varepsilon}}{|\dot{\varepsilon}|}$

(c) $\dfrac{\dot{\varepsilon} \times \ddot{\varepsilon}}{|\dot{\varepsilon} \times \ddot{\varepsilon}|}$

(d) $\dfrac{|\dot{\varepsilon} \times \ddot{\varepsilon}|}{|\dot{\varepsilon}|^3}$

(8) If $\bar{\varepsilon} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\operatorname{div} \bar{\varepsilon} =$ 1

(a) 3

(b) –2

(c) 0

(d) –1

(9) A vector $\bar{f}$ is said to be solenoidal if : 1

(a) $\operatorname{div} \bar{f} = 0$

(b) $\operatorname{curl} \bar{f} = 0$

(c) $\operatorname{grad} \bar{f} = 0$

(d) $\nabla \cdot \nabla \bar{f} = 0$

(10) A necessary and sufficient condition for $\vec{f}(t)$ to have constant magnitude is : 1

(a) $|\bar{f}| = 0$

(b) $\bar{f} \cdot \dfrac{d\bar{f}}{dt} = 0$

(c) $\bar{f} \times \dfrac{d\bar{f}}{dt} = 0$

(d) None of these

UNIT—I

2. (a) Prove that :

$$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}.$$ 4

(b) Prove that :

$$[\bar{\ell}\ \bar{m}\ \bar{n}][\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} \bar{\ell} \cdot \bar{a} & \bar{\ell} \cdot \bar{b} & \bar{\ell} \cdot \bar{c} \\ \bar{m} \cdot \bar{a} & \bar{m} \cdot \bar{b} & \bar{m} \cdot \bar{c} \\ \bar{n} \cdot \bar{a} & \bar{n} \cdot \bar{b} & \bar{n} \cdot \bar{c} \end{vmatrix}.$$ 3

(c) If $\bar{a}, \bar{b}, \bar{c}$ be three unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \dfrac{\bar{b}}{2}$, find the angles which $\bar{a}$ makes with $\bar{b}$ and $\bar{c}$, $\bar{b}$ and $\bar{c}$ being non-parallel. 3

3. (p) If $\bar{f}$ and $\bar{g}$ are vector functions of t, then prove that :

$$\frac{d}{dt}(\bar{f}\times\bar{g})=\bar{f}\times\frac{d\bar{g}}{dt}+\frac{d\bar{f}}{dt}\times\bar{g}.$$ 4

(q) If $\bar{a}=t\bar{i}-3\bar{j}+2t\bar{k}, \bar{b}=\bar{i}-2\bar{j}+2\bar{k}$ and $\bar{c}=3\bar{i}+t\bar{j}-\bar{k}$, evaluate $\int_1^2 \bar{a}\cdot(\bar{b}\times\bar{c})\,dt$. 3

(r) If $\bar{\varepsilon}=a\cos t\,\bar{i}+a\sin t\,\bar{j}+at\tan\alpha\,\bar{k}$, find $|\dot{\bar{\varepsilon}}\times\ddot{\bar{\varepsilon}}|$ and $[\dot{\bar{\varepsilon}}\,\ddot{\bar{\varepsilon}}\,\dddot{\bar{\varepsilon}}]$. 3

UNIT—II

4. (a) Prove that necessary and sufficient condition that a curve be helix is that ratio of torsion to curvature is constant. 5

(b) Prove that the position vector of the current point on a curve satisfies the differential equation :

$$\frac{d}{ds}\left(\sigma\frac{d}{ds}(\rho\,\bar{\varepsilon}^{11})+\frac{d}{ds}\left(\frac{\sigma}{\rho}\bar{\varepsilon}^{1}\right)+\frac{\rho}{\sigma}\bar{\varepsilon}^{11}=0\right).$$ 5

5. (p) State and prove Serret-Frenet formulae. 5

(q) For a point of the curve of intersection of the surfaces $x^2 - y^2 = c^2$, $y = x \tan h(z/c)$, prove that $\rho=-6=2x^2/c$. 5

UNIT—III

6. (a) Find a unit normal to the surface $xy^2 + 2yz = 4$ at the point (–2, 2, 3). 3

(b) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point (1, –2, –1). 3

(c) A vector field is given by $\bar{F}=\sin y\,\bar{i}+x(1+\cos y)\bar{j}$. Evaluate the line integral over the circular path $x^2 + y^2 = a^2$, $z = 0$. 4

7. (p) Find the work done in moving a particle in a force field given by $\bar{F}=2xy\bar{i}+3z\bar{j}-6x\bar{k}$ along the curve $x = t^2 + 1$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$. 5

(q) Let R be a closed bounded region in the xy-plane whose boundary is a simple closed curve c which may be cut by any line parallel to the coordinate axes in at most two points. Let M(xy) and N(xy) be functions that are continuous and have continuous partial derivatives $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ in R. Then prove that :

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy = \int_C (M\, dx + N\, dy)$$

where C is traversed in the positive direction. 5

UNIT—IV

8. (a) Prove that the two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ will be orthogonal if $2u_1u_2 + 2v_1v_2 + 2w_1w_2 = d_1 + d_2$. 5

(b) A sphere of radius K passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 5

9. (p) Find the coordinates of the centre and radius of the circle :

$$x + 2y + 2z = 15,\ x^2 + y^2 + z^2 - 2y - 4z = 11.$$ 5

(q) Find the equation of two spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch $4x + 3y - 15 = 0$. 5

UNIT—V

10. (a) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. 5

(b) Find the equation of the right circular cylinder of radius 4, whose axis passes through the origin and makes equal angles with the co-ordinate axes. 5

11. (p) Prove that every homogeneous equation of second degree in x, y and z represents a cone whose vertex is at the origin. 5

(q) Find the equation of right circular cone whose vertical angle is 90° and its axis is along the line $x = -2y = z$. 5