B.Sc. (Part—I) (Semester—I) Examination
MATHEMATICS
(Algebra & Trigonometry)
Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question **ONE** is compulsory. Attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :—

(1) Which **one** of the following statements is true :— 10

(a) cosh(x + iy) = coshx·cosy + isinhx·siny

(b) cosh(x + iy) = cosx cosy + isinx siny

(c) cosh (x + iy) = coshx + cosy – isinhx·siny

(d) cosh(x + iy) = coshx siny + isinhx cosy

(2) What is the value of $\sinh^{-1}x$:

(a) $\log\left[x+\sqrt{x^2+1}\right]$ (b) $\log\left[x+\sqrt{x^2-1}\right]$

(c) $\log\left[x+\sqrt{1-x^2}\right]$ (d) None of these

(3) The value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}$ is ______.

(a) π/2 (b) π/4

(c) π/3 (d) π

(4) Sum of the series $x-\frac{x^2}{2}+\frac{x^3}{3}-......+(-1)^{n+1}\frac{x^n}{n}+.....\,;\ -1<x<1$ is denoted by ____.

(a) log(1 + x) (b) sinhx

(c) coshx (d) e^x

(5) If q = 2 + 2i –j + 4k then the norm of q is _____.

(a) –5 (b) 5

(c) 1/5 (d) None of these

(6) The inverse of unit quaternion is its ____.

(a) Purely imaginary (b) Purely real

(c) Complex conjugate (d) None of these

(7) If α + iβ be the root of quadratic polynomial f(x) = 0 then its another root is _____.

(a) α – iβ (b) α

(c) β (d) None of these

(8) If α, β, γ are the roots of the equation $px^3 + qx^2 + rx + s = 0$ then Σα is _____.

(a) $\frac{q}{p}$ (b) $-\frac{q}{p}$

(c) $\frac{r}{p}$ (d) $\frac{s}{p}$

(9) If A and B are the non-signular matrices of order n then _____.

(a) $(AB)^{-1} = AB$ (b) $(AB)^{-1} = A^{-1} \cdot B^{-1}$

(c) $(AB)^{-1} = B^{-1} \cdot A^{-1}$ (b) None of these

(10) 'Every square matrix satisfies its own characteristics equation' is the statement of _____.

(a) Lagrange's MVT (b) De-Moivre's theorem

(c) Cayley-Hamilton theorem (d) Cauchy's MVT

UNIT—I

2. (a) Prove that $\dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta} = \sin\theta + i\cos\theta$.

Hence prove that $\left(1+\sin\frac{\pi}{5}+i\cos\frac{\pi}{5}\right)+i\left((1+\sin\frac{\pi}{5}-i\cos\frac{\pi}{5}\right)=0$. 5

(b) If $\sin(\alpha+i\beta) = x+iy$ then prove that $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\sinh^2\beta} = 1$ and $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha} = 1$ 5

3. (p) Prove that one of the value of :
$(\cos\theta + i\sin\theta)^n$ is $(\cos n\theta + i\sin n\theta)$; when n is negative integer. 5

(q) Separate real and imaginary parts of $\tan(x + iy)$. 5

UNIT—II

4. (a) Find the Sum of the series :

$$C = 1 + e^{\sin x} \cdot \cos(\cos x) + \frac{1}{2!}e^{2\sin x} \cdot \cos(2\cos x) + e^{3\sin x} \cdot \frac{1}{3!}\cos(3\cos x) + ...$$ 5

(b) Prove that $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$. 5

5. (p) Find the sum of the series $\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + ...$ 5

(q) If $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ then prove that

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + ...+(-1)^{n-1}\frac{1}{2n-1}\tan^{2n-1} x + ...$$ 5

UNIT—III

6. (a) Prove that for p, q∈H, N(pq) = N(p) N(q) and N(q*) = N(q). 5

(b) For the quaternion $q = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ and the input vector v = i, compute the output vector w under the action of the operators L_q and L_{q^*}. 5

7. (p) Show that the quaternian product need not be commutative. 5

(q) For any p, q∈H, show that pq = qp if and only if p and q are parallel. 5

UNIT—IV

8. (a) Find the roots of the equation, $8x^3 + 18x^2 - 27x - 27 = 0$, if these roots are in geometric progression. 5

   (b) State Descartes rule of sign. Find the nature of the roots of the equation $2x^7 - x^4 + 4x^3 - 5 = 0$. 5

9. (p) Prove that in an equation with real coefficients complex roots occur in pairs. 5

   (q) Solve the equation $x^4 - 2x^3 - 22x^2 + 62x - 15 = 0$; given that $2\sqrt{3}$ is one of the root. 5

UNIT—V

10. (a) Show that if λ is the eigen value of a nonsingular matrix A then λ^{-1} is the eigen value of A^{-1}. 5

    (b) Find the eigen values and the corresponding eigen vector for smallest eigen value of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. 5

11. (p) Show that the eigen values of any square matrix A and A' are same. 5

    (q) Reduce to canonical form and find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 5

B.Sc. Part-I (Semester-I) Examination

MATHEMATICS

(Differential & Integral Calculus)

Paper—II

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (**1** mark each) : 10

(i) The value of $\lim_{x\to 0} \frac{\sin x}{x}$ is :

(a) 0 (b) 1

(c) ∞ (d) None of these

(ii) If $y = e^{-2x}$, then y_{11} is :

(a) $-2^{11} e^{-2x}$ (b) $2^{11} e^{-2x}$

(c) $-2^{11} e^{2x}$ (d) None of these

(iii) The series :

$$x - \frac{x^3}{3!} + \ldots\ldots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \ldots\ldots$$

is the expansion of function :

(a) sin x (b) sinh x

(c) cos x (d) cosh x

(iv) $|x - x_0| < \delta$ represents :

(a) $x_0 - \delta < x < x_0 + \delta$ (b) $x_0 + \delta < x < x_0 - \delta$

(c) $x_0 - \delta \le x < x_0 + \delta$ (d) $x_0 - \delta < x \le x_0 + \delta$

(v) If f be differentiable on (a, b) and $f'(x) = 0$, $\forall$ $x \in [a, b]$, then f(x) is :

(a) Monotonic increasing in [a, b] (b) Monotonic decreasing in [a, b]

(c) Constant in [a, b] (d) None of these

(vi) For $f(x) = x^2$; in [1, 3] then the value of 'C' by Lagrange's mean value theorem is :

(a) $\frac{6}{13}$ (b) 2

(c) 0 (d) 1

(vii) The area bounded by the curve $x = g(y)$; y-axis and $y = a$, $y = b$ is :

(a) $\int_a^b y\,dx$ (b) $\int_a^b x\,dy$

(c) $\int_a^b y^2\,dx$ (d) $\int_a^b x^2\,dy$

(viii) The functions f and g be :

(i) continuous in [a, b]

(ii) derivable in (a, b) and

(iii) $g'(x) \neq 0$ for all $x \in (a, b)$.

These are the hypothesis of mean value theorem by :

(a) Rolle's (b) Lagrange's

(c) Cauchy's (d) Leibnitz

(ix) The function f(x) has the removable discontinuity if :

(a) $f(x^+) \neq f(x^-)$ (b) $f(x^+) = f(x^-) \neq f(x)$

(c) $f(x^+)$, $f(x^-)$ do not exist (d) None of these

(x) $\frac{d}{dx}$ cosh x is :

(a) sinh x (b) –sinh x

(c) h sinh x (d) –h sinh x

UNIT—I

2. (a) If $\lim_{x \to x_0} f(x) = \ell$ and $\lim_{x \to x_0} g(x) = m$, then prove that :

$$\lim_{x \to x_0} [f(x) + g(x)] = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x)$$

$$= \ell + m.$$ 4

(b) Prove that the function defined by $f(x) = x^2$ is continuous for all $x \in R$. 3

(c) Using definition of limit, prove that :

$$\lim_{x \to 3} \frac{x^3 - 2x^2 - x - 6}{x - 3} = 14.$$ 3

3. (p) Define limit of a function and show that the limit of a function if it exist, is unique. 1+3

(q) Prove that $\lim_{x \to 2} x^2 = 4$; by using $\in$-δ definition. 3

(r) If $f(x) = \frac{e^{1/x}}{1+e^{1/x}}, x \neq 0,$

$= 0, \quad , x = 0$

then show that f(x) has a simple discontinuity at x = 0. 3

UNIT—II

4. (a) Prove that if f(x) is differentiable at $x = x_0$, then it is continuous at $x = x_0$. Is converse of this statement true ? Justify. 5

(b) Evaluate :

$$\lim_{x\to 0} (\cos x)^{\cot^2 x}.$$ 3

(c) If y = A sin mx + B cos mx, then prove that $y_2 + m^2y = 0$. 2

5. (p) If $y = \sin(m \sin^{-1}x)$, then show that :

(i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$ 5

(q) If $y = \frac{1}{ax+b}$, then prove that $y_n = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$. 3

(r) Evaluate :

$$\lim_{x\to 0} \frac{x - \sin x}{x^3}.$$ 2

UNIT—III

6. (a) State and prove Lagrange's mean value theorem. 4

(b) Verify Cauchy mean value theorem for the functions :

$f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]. 3

(c) Expand sin x in powers of $x - \frac{\pi}{2}$, upto first four terms. 3

7. (p) State and prove Cauchy's mean value theorem. 4

(q) Expand $3x^3 + 4x^2 + 5x - 3$ about the point x = 1 by Taylor's theorem. 3

(r) Verify the Rolle's theorem for the function :

$$f(x) = \frac{\sin x}{e^x} \text{ in } [0, \pi].$$ 3

UNIT—IV

8. (a) If u = f(x, y, z) is a homogeneous function of degree n, then show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu.$$ 4

(b) Verify Euler's theorem for $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$. 3

(c) If $u = e^x (x \cos y - y \sin y)$, then find the value $u_{xx} + u_{yy}$. 3

9. (p) If $u = f(x, y)$ be homogeneous function of degree n then prove that :

(i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$ are homogeneous functions of degree 'n − 1' in x, y and

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$. 4

(q) If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$, then show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$ 3

(r) If $u = \log \frac{x^4 - y^4}{x - y}$, $x \neq y$, then prove that :

(i) $x u_x + y u_y = 3$

(ii) $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = -3$. 3

UNIT—V

10. (a) Prove that :

$$\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, dx.$$ 4

(b) Evaluate :

$$\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}} dx.$$ 3

(c) Show that '8a' is the length of an arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$; $0 \leq t \leq 2\pi$. 3

11. (p) Prove that :

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}.$$

Hence evaluate $\int \tan^3 x \, dx$. 4

(q) Find the area bounded by the x-axis, the curve $y = c \cosh \frac{x}{c}$ and the ordinates $x = 0$, $x = a$. 3

(r) Show that length of the curve $y = \log \sec x$ between the points, where $x = 0$ and $x = \frac{\pi}{3}$ is $\log_e (2 + \sqrt{3})$. 3

B.Sc. (Part-I) Semester-I Examination

MATHEMATICS

(New Course)

(Algebra and Trigonometry)

Paper—I

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The value of $(\cos\theta - i\sin\theta)^{-n}$ is : 1

(a) $\cos n\theta + i\sin n\theta$ (b) $\cos n\theta - i\sin n\theta$

(c) $\sin n\theta + i\cos n\theta$ (d) $\sin n\theta - i\cos n\theta$

(ii) The value of sin(iz) is : 1

(a) sinh z (b) i sinh z

(c) i sin z (d) sin z

(iii) If $x - n\pi = \tan x - \frac{1}{3}\tan^3 x + \ldots\ldots$, then the value of n when x lies between $-\frac{3\pi}{4}$ and $-\frac{5\pi}{4}$ is : 1

(a) 1 (b) –1

(c) 0 (d) None of these

(iv) The value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$ is : 1

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(v) Hamilton product $\vec{i}\,\vec{j}\,\vec{k} = \ldots\ldots\ldots$ 1

(a) 1 (b) −1

(c) 0 (d) None of these

(vi) If f is selection function, $f(2 + 3\vec{i} + \vec{j} - \vec{k}) = \ldots\ldots\ldots$ 1

(a) 2 (b) 3

(c) 1 (d) −1

(vii) Every equation of degree n has : 1

(a) n roots (b) more than n roots

(c) less than n roots (d) None of these

(viii) The Descartes rule of signs does not tell about the : 1

(a) Positive root of equation (b) Negative root of equation

(c) Zero root of equation (d) None of these

(ix) Elementary transformations : 1

(a) affect the rank of a matrix

(b) do not affect the rank of matrix

(c) have the different rank of a matrix

(d) None of these

(x) An n-square matrix A has rank $r < n$ iff : 1

(a) det (A) = 0 (b) det (A) ≠ 0

(c) det (A) = ∞ (d) None of these

UNIT—I

2. (a) By using DeMoivre's theorem, find all the fourth root of 81. 5

(b) If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = z^{n+1} \cdot \cos\frac{n\pi}{3}$. 5

3. (p) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that :

$\cos^2\theta = \pm \sin\alpha$. 5

(q) Separate into real and imaginary parts of $\tan(x + iy)$. 5

UNIT—II

4. (a) Prove that :

$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}.$$ 4

(b) Sum the series :

$$\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + \ldots\ldots\ldots$$ 6

5. (p) If $x < \sqrt{2} - 1$ then prove that :

$$2\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \ldots\ldots\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^2 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^3.$$ 4

(q) Sum the series :

$$a\cos x - \frac{1}{3}a^3\cos(x+2y) + \frac{1}{5}a^5\cos(x+4y) - \ldots\ldots\ldots$$ 6

UNIT—III

6. (a) Show that quaternian product need not be commutative. 4

(b) Prove that for $p, q \in H$, $N(pq) = N(p)\,N(q)$ and $N(q^*) = N(q)$. 4

(c) Show that the quaternian product of two vectors $\vec{r}$ and $\vec{s}$ is given by $\vec{r}\vec{s} = \vec{r} \times \vec{s} - \vec{r} \cdot \vec{s}$. 2

7. (p) If $Lq(\vec{v}) = q\,\vec{v}\,q^*$, then prove that $f(Lq(\vec{v})) = 0$ and hence show that $Lq(\vec{v}) \in R^3$. 4

(q) If q is a unit quarterian and $\vec{v} = k\vec{q}$, where $k \in R$, then show that the output vector $\vec{w} = Lq(\vec{v}) = k\vec{q}$. 4

(r) Write the quaternian inverse for $q = a\cos\theta - b\,\vec{u}\sin\theta$. 2

UNIT—IV

8. (a) Solve by Cardon's method $x^3 - 15x = 126$. 5

(b) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$, roots being in A.P. 5

9. (p) State Descartes' rule of sign. Find the nature of the roots of the equation $2x^7 - x^4 + 4x^3 - 5 = 0$. 1+4

(q) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. 5

UNIT—V

10. (a) Reduce the matrix $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ to the normal form and then find its rank. 5

(b) Prove that the eigenvalues of a Hermitian matrix are all real. 5

11. (p) State Cayley-Hamilton theorem. Verify it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. 1+4

(q) Find the eigenvalues and the corresponding eigenvectors of the matrix :

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$ 5

B.Sc. (Part—I) (Semester—I) Examination
MATHEMATICS
(Algebra and Trigonometry)
Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(i) The period of sinh z is :

(a) $2\pi i$ (b) πi

(c) $\frac{\pi}{2}i$ (d) i 1

(ii) The value of $e^{-\frac{\pi}{2}i}$ is :

(a) –i (b) 1 + i

(c) 1 – i (d) 0 1

(iii) The series $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$ is called :

(a) Gregory's series (b) Euler's series

(c) Rutherford's series (d) Machiri's series 1

(iv) The sum of infinite Geometric series $a + ar + ar^2 \ldots\ldots + ar^{n-1} + \ldots\ldots\ldots, |r| < 1$ is :

(a) $\frac{d}{1-r}$ (b) $\frac{r}{1-r}$

(c) $\frac{r}{a-r}$ (d) .1 1

(v) The norm of quaternion $q = 2 + 2\vec{i} - \vec{j} + 4\vec{k}$ is :

(a) 2 (b) 9

(c) 4 (d) 5 1

(vi) For any quaternion q, its inverse is equal to :

(a) –q (b) q*

(c) –q* (d) None of these 1

(vii) The polynomial of fourth degree is called as :

(a) Linear (b) Quadratic

(c) Biquadratic (d) Cubic 1

(viii) The degree of an equation having roots $(3 + i)$ is :

(a) 1 (b) 2

(c) 3 (d) 4 1

(ix) The rank of zero matrix is :

(a) 1 (b) 0

(c) n (d) None of these 1

(x) The number of positive and negative roots of an equation of degree n is found by :

(a) Cardan's Method (b) Ferrari's Method

(c) Descartes' rule of signs (d) None of these 1

UNIT—I

2. (a) State DeMoivre's theorem and prove it for positive integer. 1+4

(b) Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos\left(\frac{n\pi}{4}\right)$, where n being positive integer. 5

3. (p) Separate the following expression into real and imaginary parts :

(i) $\sinh(x + iy)$

(ii) $\tan(x + iy)$. 4

(q) Find all the value of $(-1)^{1/3}$. 3

(r) Show that $\cosh^{-1} x = \log\left\{x + \sqrt{x^2 - 1}\right\}$. 3

UNIT—II

4. (a) If $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$, then prove that :

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x \,..... + (-1)^{n-1}\frac{\tan^{2n-1} x}{2n-1} + ...$$ 5

(b) Sum the series :

$$S = a \sin x + \frac{1}{2}a^2 \sin 2x + \frac{1}{3}a^3 \sin 3x + ...$$ 5

5. (p) Prove that :

$$\frac{\pi}{4} = \frac{1}{2} - \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + ... + \frac{1}{3} - \frac{1}{3}\cdot\frac{1}{3^3} + \frac{1}{5}\cdot\frac{1}{3^5} ......$$ 5

(q) Sum the series :

$$a \sin x - \frac{1}{3}a^3 \sin 3x + \frac{1}{5}a^3 \sin 5x - ....$$ 5

UNIT—III

6. (a) If $p = 2 - 3\vec{i} - 4\vec{j} + 5\vec{k}$ and $q = -6 + \vec{i} + 2\vec{j} - 3\vec{k}$, then find the quaternion product pq. 4

(b) Show that :

$pq = qp \Leftrightarrow \overline{p}$ and $\overline{q}$ are parallel, for some $p, q \in H$. 4

(c) Prove that quaternion product $\vec{i} \cdot \vec{j} = \vec{k}$. 2

7. (p) Let q is any unit quaternion, then prove that :

$$Lq(\vec{v}) = \vec{w} = (q_0 - |\vec{q}|^2)\vec{v} + 2(\vec{q} \cdot \vec{v})\vec{q} + 2q_0(\vec{q} \times \vec{v})$$ 5

(q) If the quaternion $q = \cos\frac{\pi}{3} + i \sin\frac{\pi}{3}$ and input vector $\bar{v} = i$, then compute the output vector $\vec{w}$ under the action of operator Lq. 5

UNIT—IV

8. (a) Find the equation whose roots are the roots of equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. 4

(b) If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in G.P. Prove that $a^3c = b^3$. 3

(c) Find the equation whose roots are the reciprocals of $x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$. 3

9. (p) Solve the equation $x^3 - 21x = 344$ by Cardan's method. 4

(q) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the values of :

(i) $\Sigma\alpha^2$

(ii) $\Sigma\alpha^2\beta$. 4

(r) Show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. 2

UNIT—V

10. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 5

(b) Verify Cayley-Hamilton theorem for matrix A :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$ 5

11. (p) Find the row rank and column rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$. 5

(q) Show that the eigen values of Hermitian matrix are all real. 5

AS–1346

B.Sc. (Part—I) Semester—I Examination
MATHEMATICS (New)
Paper—I
(Algebra and Trigonometry)

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(i) If i, 1 + i are the roots of $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$ then remaining roots are :

(a) – i, 1 – i (b) – i, i – 1

(c) i, 1 – i (d) – i, – 1 – i 1

(ii) The real part of sin (x + iy) is :

(a) sin x · cosh y (b) cos x · sinh y

(c) sin x · sinh y (d) cos x · cosh y 1

(iii) The series $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5}$........... is called as :

(a) Euler's series (b) Gregory's series

(c) Rutherford series (d) Geometric series. 1

(iv) If $\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta$ – .... then the values of n when θ lies between $\frac{19\pi}{4}$ & $\frac{21\pi}{4}$ is :

(a) n = 4 (b) n = – 4

(c) n = 5 (d) n = –5 1

(v) The unit quaternion has :

(a) Both real and vector part one

(b) Both real and vector part zero

(c) real part one and vector part zero

(d) None of these. 1

(vi) The norm of quaternion $q = 2 + 2\bar{i} - \bar{j} + 4\bar{k}$ is :

(a) 5 (b) 4

(c) 2 (d) 9 1

(vii) The number of positive and negative roots of an equation of degree n is found by :

(a) Cardon's method (b) Ferrari's method

(c) Descarte's rule of signs (d) None of these 1

(viii) An equation of four degree is called as :

(a) Linear (b) Quadratic

(c) Cubic (d) Biquadratic 1

(ix) The rank of a zero matrix is :

(a) 1 (b) 0

(c) n (d) None of these 1

(x) If the matrix is n-square identity matrix then its rank is :

(a) n (b) 1

(c) 0 (d) None of these 1

UNIT—I

2. (a) State DeMoivre's theorem and prove it for positive integer. 1+4

(b) Find n, n^{th} roots of unity and show that they form a series in G.P. 5

3. (p) Prove that $\cosh^{-1} x = \log\left[x + \sqrt{x^2 - 1}\right]$. 5

(q) If $\sin(\alpha + i\beta) = x + iy$, then prove that $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$ and $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$. 5

UNIT—II

4. (a) If $\frac{-\pi}{4} \le x \le \frac{\pi}{4}$, then show that :

$$x = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x - .... + (-1)^{n-1}\frac{1}{2n-1}\tan^{2n-1} x + ...$$ 5

(b) Sum the series $\sinh x + \frac{1}{2!}\sinh 2x + \frac{1}{3!}\sinh 3x + ...$ 5

5. (p) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. 5

(q) Sum the series $\cos x - \frac{1}{3!}\cos(x+2y) + \frac{1}{5!}\cos(x+4y) - ......$ 5

UNIT—III

6. (a) Show that quaternion product need not be commutative. 5

(b) Prove that for any p, q ∈ H, $(pq)^* = q^* p^*$. 5

7. (p) Show that for any p, q ∈ H, pq = qp iff $\vec{p}$ and $\vec{q}$ are parallel. 5

(q) Define inverse of a quaternion. Show that for any nonzero quaternion q, $q^{-1} = \frac{q^*}{N^2(q)}$. 1+4

UNIT—IV

8. (a) Prove that in a polynomial equation with real coefficients, complex roots occur in pairs. 5

(b) Solve the equation $x^3 - 15x = 126$ by Cardon's method. 5

9. (p) State Descarte's rule of signs and show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. 1+4

(q) Find the condition that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P. 5

UNIT—V

10. (a) Define a row rank and column rank of a matrix. Show that row rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$ is 2. 1+1+3

(b) State Caley-Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. 1+4

11. (p) Define eigenvalues and eigenvectors of a matrix. If λ is an eigenvalue of matrix A, then show that λ^m is an eigenvalue of the matrix A^m, for any positive integral value of m. 1+1+3

(q) Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. 5

First Semester B. Sc. (Part – I) Examination

(New)

MATHEMATICS

Paper – I

(Algebra and Trigonometry)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **One** is compulsory and attempt it once only.

(2) Attempt **One** question from each unit.

1. Choose the correct alternative :—

(i) If $z = 1 + i\sqrt{3}$, then $|Z|$ is

(a) 0

(b) 1

(c) 2

(d) 3. 1

(ii) The value of $e^{-\pi i}$ is

(a) 0

(b) – 1

(c) i

(d) 3. 1

(iii) If $\theta - n\pi = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta$ ..... , then the value of n when θ lies between

$\frac{7\pi}{4}$ and $\frac{9\pi}{4}$ is

(a) n = 2

(b) n = − 2

(c) n = 3

(d) n = − 3. 1

(iv) The series $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$ is called

(a) Gregory's series

(b) Euler's series

(c) Rutherford's series

(d) Machin's series. 1

(v) The norm of quaternion $q = 5 + \overline{2i} - \overline{4j} + \overline{2k}$ is

(a) 4

(b) 5

(c) 6

(d) 7. 1

(vi) The identity quaternion has

(a) both real and vector part zero

(b) both real and vector part one

(c) real part one and vector part zero

(d) none of these. 1

(vii) The equation $(x^2 + 5)^2 = 0$ must have

(a) Two roots

(b) Three roots

(c) Four roots

(d) Five roots. 1

(viii) The equation with integral coefficients having a root $-2 + \sqrt{3}$ is :

(a) $x^2 - 4x + 1 = 0$

(b) $x^2 + 4x + 1 = 0$

(c) $x^2 - 4x - 1 = 0$

(d) $x^2 + 4x - 1 = 0$. 1

(ix) For a symmetric matrix the eigen vectors are

(a) equal

(b) orthogonal

(c) parallel

(d) none of these. 1

(x) Every square matrix A satisfies its own characteristic equation. This is :

(a) De – Moivre's theorem

(b) Euler's theorem

(c) Cayley – Hamilton theorem

(d) None of these. 1

UNIT – I

2. (a) Show that the continued product of four values of $\left(\cos\frac{\pi}{3} - i\ \sin\frac{\pi}{3}\right)^{3/4}$ is unity. 3

(b) If $\sin(\alpha + i\beta) = x + iy$, prove that

(i) $$\frac{x^2}{\cos h^2 \beta} + \frac{y^2}{\sin h^2 \beta} = 1$$

(ii) $$\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$$

4

(c) Prove that $\sin h^{-1} x = \log\{x + \sqrt{x^2 + 1}\}$. 3

3. (p) State De - Moivre's theorem. Prove it for negative integers. 1 + 4

(q) Separate in to real and imaginary parts of $\tan^{-1}(x + iy)$. 5

UNIT – II

4. (a) Prove that

$$\frac{\pi}{4} = 4\left[\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5}\ldots\right] - \left[\frac{1}{23g} - \frac{1}{3}\cdot\frac{1}{23g^3} + \frac{1}{5}\cdot\frac{1}{23g^5}\ldots\right]$$

5

(b) Sum the series

$$a\cos x - \frac{1}{3}a^3\cos(x + 2y) + \frac{1}{5}\cdot a^5\cos(x + 2y) + \ldots\ldots$$ 5

5. (p) Prove that

$$\frac{\pi}{2\sqrt{3}} = \left[1 - \frac{1}{3^2} + \frac{1}{5}\cdot\frac{1}{3^2} - \frac{1}{7}\cdot\frac{1}{3^3} + \ldots\ldots\right]$$

5

(q) Sum the series

$$\sin h\,x + \frac{1}{2!}\sin h2x + \frac{1}{3!}\sin h3x + \ldots\ldots$$

5

UNIT – III

6. (a) If $p = 2 - \overline{i} + \overline{3j} - \overline{4k}$ and $q = 5 + \overline{2i} - \overline{4j} + \overline{3k}$, then find the quaternion product pq. 5

   (b) Show that quaternion product is associative. 5

7. (p) Prove that for any quaternion $\overline{p}, \overline{q} \in H$
   $(pq)^* = q^* p^*$. 5

   (q) Define : Inverse of the quaternion. Show that for any non zero quaternion q.
   $$q^{-1} = \frac{q^*}{N^2(q)}$$ 5

UNIT – IV

8. (a) Prove that an equation with real coefficient complex roots occur in pair. 4

   (b) Find the condition that the roots of the polynomial equation $x^3 - ax^2 + bx - c = 0$ are in A. P. 3

   (c) State Descarte's rule of signs and find the nature of the roots of equation $3x^4 + 12x^2 + 5x - 4 = 0$. 3

9. (p) Solve the equation $x^3 - 15x - 126 = 0$ by Cardan's method. 5

(q) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. 5

UNIT – V

10. (a) Find the row rank and column–rank of matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$$

2 + 2

(b) Show that if B is an inverse matrix of the same order as A, then the matrices A and $B^{-1}AB$ have the same characteristic roots. 3

(c) Verify Cayley – Hamilton theorem for the matrix.

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

3

11. (p) Find the eigen values and eigen vectors corresponding to the highest eigen value of

matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5

(q) Show that the eigen values of a Hermitian matrix are all real 5

B.Sc. (Part—I) Semester—I Examination
MATHEMATICS
Paper—II
(Differential & Integral Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **one** question from each unit.

1. Choose the correct alternatives (1 mark each) :— 10

(i) Let $f(x) = \sin\frac{1}{x},\ x \neq 0$

$= 0 \quad , x = 0$

Then f(x) has discontinuity of _______ at x = 0.

(a) Type-II (b) Ordinary

(c) Removable (d) None of these

(ii) Let f(x) = [x] = greatest positive integer not greater than x,

then $\lim_{x\to 2} f(x) =$

(a) 0 (b) 1

(c) 2 (d) does not exist

(iii) If $y = (2x-3)^4$ then y_3 =_______.

(a) 192 (b) (2x –3)

(c) 192(2x –3) (d) 0

(iv) A function f(x) has a derivative at $x = x_0$ iff ________.

(a) $f'(x_0^+) = f'(x_0^-)$ (b) $f'(x_0^+) \neq f'(x_0^-)$

(c) $f'(x_0^+) = f'(x_0^-) \neq f'(x_0)$ (d) None of these

(v) If a real function f defined on [a, b] is :

(1) Continuous on [a, b]

(2) Differentiable on (a, b)

then there is at least one point c ∈ (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. It is statement of_____.

(a) Rolle's theorem (b) Lagrange's mean value theorem

(c) Cauchy mean value theorem (d) None of these

(vi) The series of f(x) = sin x is :

(a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....$ (b) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .....$

(c) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ (d) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...$

(vii) If $f(x, y) = x^2 + 2xy + y^2$ then f_{xy} = ________.

(a) 1 (b) 2

(c) 3 (d) 4

(viii) If $f(x, y) = \frac{1}{x} + \frac{\log x - \log y + 7}{y}$ then f(x, y) is homogeneous of degree ______.

(a) 1 (b) –1

(c) 2 (d) –2

(ix) Let f(x) be continuous and non-negative on [a, b]. Then the area A bounded by the curve y = f(x), the x-axis and two ordinates x = a, x = b is A = ________.

(a) $\int_b^a y\, dx$ (b) $\int_a^b y\, dx$

(c) $\int_a^b x\, dy$ (d) $\int_b^a x\, dy$

(x) The process of finding the length of arc of a curve by definite integral is known as :

(a) Quadrature (b) Unification

(c) Rectification (d) None of these

UNIT—I

2. (a) If $\lim_{x \to x_0} f(x) = \ell$, then f is bounded on some deleted neighbourhood of x_0, prove this. 3

(b) Show that $\lim_{x \to 1} \frac{2x^3 - x^2 - 8x + 7}{x - 1} = -4$. 3

(c) Discuss the continuity of the function $f(x) = (x - a) \sin \frac{1}{(x - a)}, \quad x \neq a$

$= 0 \quad , x = a.$

at point x = a. 4

3. (p) If $\lim_{x \to x_0} f(x)$ exists, then it is unique. Prove this. 4

(q) Show that $\lim_{x \to 0} f(x)$ does not exist, if $f(x) = \begin{cases} |x|/x & , x \neq 0 \\ 0 & , x = 0 \end{cases}$. 3

(r) Let $f(x) = \begin{cases} \frac{\sin}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, show that f(x) has removable discontinuity at x = 0. 3

UNIT—II

4. (a) Show that $f(x) = x^2$ is differentiable in $0 \leq x \leq 2$. 3

(b) Find y_n for $y = \tan^{-1}\left(\frac{x}{a}\right)$. 3

(c) If $y = x^n \cdot \log x$, then show that $y_{n-1} = \frac{n!}{x}$. 4

5. (P) Prove that $\lim_{x\to 1}\left[\frac{1}{\log x}-\frac{x}{x-1}\right]=-\frac{1}{2}$. 3

(q) If $y = \cos x \cdot \cos 2x \cdot \cos 3x$, find y_n. 3

(r) If $y = e^{a\sin^{-1}x}$, prove that :
$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n=0$. 4

UNIT—III

6. (a) If f and g are continuous real functions on [a, b] which are differentiable in (a, b), then there is a point $c\in(a, b)$ such that $\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$, where $g(a) \neq g(b)$ and $f'(x)$, $g'(x)$ are not simultaneously zero. 4

(b) Verify Lagrange's mean value theorem for $f(x) = \log x$ in $[1, e]$. 3

(c) Expand sin x in power of $(x - \frac{1}{2}\pi)$. 3

7. (p) Verify the truth of Rolle's theorem for $f(x) = x^2 + x - 6$ in $[-3, 2]$. 4

(q) Expand $\tan^{-1}x$ in powers of $\left(x-\frac{\pi}{4}\right)$. 3

(r) If f is differentiable on (a, b) and $f'(x) \geq 0$, $\forall\, x\in(a, b)$ then prove that f is monotone increasing on (a, b). 3

UNIT—IV

8. (a) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$, show that $u_{xx} + u_{yy} + u_{zz} = 0$. 3

(b) If $u = f(x, y)$ is a homogeneous differentiable function of degree n in x, y then $xu_x + yu_y = nu$. Prove this. 4

(c) If $Z=f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then show that :

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2.$$ 3

9. (p) If $u = f(x + ay) + g(x - ay)$, show that $u_{yy} = a^2u_{xx}$. 3

(q) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, then show that $x^2u_{xx}+ 2xyu_{xy} + y^2u_{yy}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{2}\right)$. 4

(r) If $z = f(x^2- y^2)$, show that $yz_x + xz_y = 0$. 3

UNIT—V

10. (a) Find the value $\int_0^1 \frac{1-4x+2x^2}{\sqrt{2x-x^2}}dx.$ 3

(b) Prove that $\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$, n is even.

$= \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\cdots\frac{2}{3}$, n is odd. 4

(c) Calculate the area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ 3

11. (p) If $I_n = \int \sin^n x\,dx$ then $I_n = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\,I_{n-2}$. 4

(q) Find the length of the arc of the equiangular spiral $r = ae^{\theta \cot\alpha}$ between the points for which the radii vectors are r_1 and r_2. 3

(r) Integrate $\int \frac{x^2+2x+3}{\sqrt{x^2+x+1}}dx.$ 3

B.Sc. (Part-I) Semester-I Examination

(New Course)

1S : MATHEMATICS

Paper—II

(Differential & Integral Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (1 mark each) : 10

(i) If $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$ and f(x) is defined at x = a then which type of discontinuity occurs :

(a) First kind (b) Second kind

(c) Removable (d) None of these

(ii) Which of the following function is continuous at origin ?

(a) $f(x) = \cos(1/x)$, when $x \neq 0$ and $f(0) = 0$

(b) $f(x) = \sin(1/x)$, when $x \neq 0$ and $f(0) = 0$

(c) $f(x) = x + \sin(1/x)$, when $x \neq 0$ and $f(0) = 1$

(d) $f(x) = x \cdot \sin(1/x)$, when $x \neq 0$ and $f(0) = 1$

(iii) If $y = e^{-3x}$ then $y_{11} = ?$

(a) $-3^{11}e^{-3x}$ (b) $3^{11}e^{-3x}$

(c) $-e^{-3x}$ (d) None of these

(iv) If f(x) is defined and continuous on [a, b]; derivable on (a, b) then there exist at least one point $c \in (a, b)$ such that $f(b) - f(a) = (b - a) f'(c)$ which is the statement of :

(a) Lagranges mean value theorem (b) Rolle's theorem

(c) Cauchy's mean value theorem (d) Intermediate value theorem

(v) If $f(x) = x^2 + x - 6$; $x \in [-3, 2]$ then the value of 'c' by Rolle's theorem is :

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) 1

(vi) For $f(x) = x^2$; $g(x) = x^3$ in [1, 3] then the value of 'c' by Cauchy's mean value theorem is :

(a) $\frac{6}{13}$ (b) $\frac{13}{6}$

(c) 0 (d) 1

(vii) If $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots$ then f(x) is :

(a) $\log(1 + x)$ (b) $\sin x$

(c) $\cos x$ (d) $\tan^{-1}x$

(viii) The value of $\lim_{x \to 0} (x^x)$ is :

(a) e (b) 1/e

(c) 0 (d) 1

(ix) What is the value of $\lim_{x \to \infty} \left[\frac{2^{n+1} + 3^{n+1}}{2^n + 3^n} \right]$?

(a) $\frac{1}{3}$ (b) 1

(c) 3 (d) 0

(x) The area bounded by the curve $x = g(y)$; y-axis and $y = a$, $y = b$ is :

(a) $\int_a^b y\,dx$ (b) $\int_a^b x\,dy$

(c) $\int_a^b y^2\,dx$ (d) None of these

UNIT—I

2. (a) Prove that limit of function, if it exist, then it is unique. 4

(b) Show that the function $f(x) = x \cdot \sin\left(\frac{1}{x}\right); x \neq 0$ is continuous at x = 0. 3

$= 0$; otherwise

(c) If f(x) is defined and continuous in [a, b] then prove that f(x) attain every value between its bounds. 3

3. (d) Prove that limit of product of two functions is equal to the product of their limits. 4

(e) Show that the function $f(x) = (1+2x)^{1/x}; x \neq 0$ is continuous at x = 0. 3

$= e^2 \quad ; x = 0$

(f) Using $\in-\delta$ definition, prove that :

$$\lim_{x\to 3}\left(\frac{1}{x}\right) = \frac{1}{3}.$$ 3

UNIT—II

4. (a) State and prove Leibnitz theorem. 5

(b) Evaluate $\lim_{x\to 0}\left[\frac{e^x - e^{-x} - 2\log(1+x)}{x \cdot \sin x}\right]$. 3

(c) Find y_n, if $y = (ax + b)^{-1}$. 2

5. (d) If $y = (x + \sqrt{x^2 - 1})^m$ then prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 4

(e) If $y = \frac{x^3}{x^2 - 1}$ then find (y_n) at x = 0. 3

(f) Prove that $\lim_{x\to\infty}\left[\frac{\pi}{2} - \tan^{-1}x\right]^{1/x} = 1$. 3

UNIT—III

6. (a) State and prove Lagranges mean value theorem. 5

(b) Verify Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]. 3

(c) Expand e^x upto first four terms at x = 0. 2

7. (d) If f(x) and g(x) are continuous real valued functions on [a, b]; which are differentiable in (a, b) then prove that there exist at least one point 'c' in (a, b) such that :

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}; \text{ where } g(a) \neq g(b).$$ 4

(e) Expand $2x^3 + 7x^2 + x - 1$ in powers of (x − 2). 3

(f) Verify Rolle's theorem for $f(x)=\log\left[\frac{x^2+ab}{(a+b)x}\right]$ in [a, b]; $x \neq 0$. 3

UNIT—IV

8. (a) State and prove Euler's theorem for function of two variables. 4

(b) If $u=\frac{x^2+y^2}{x+y}$ then prove that $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left(1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)$. 3

(c) If $u=\frac{1}{\sqrt{x^2+y^2+z^2}}; x^2+y^2+z^2 \neq 0$; then show that $u_{xx} + u_{yy} + u_{zz} = 0$. 3

9. (d) If F(u) be a homogeneous function of degree 'n' in x and y, where u is a function of x, y then prove that :

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n\frac{F(u)}{F'(u)}$ and

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=G(u)[G'(u)-1]$;

where $G(u) = nF(u) / F'(u)$. 5

(e) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z} \text{ and}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{9}{(x+y+z)^2}.$$ 3

(f) If $f(x, y) = 2x^3y^2 - 3xy^2 + x - 2y$ then prove that $\frac{\partial^2 f}{\partial x \partial y}=\frac{\partial^2 f}{\partial y \partial x}$. 2

UNIT—V

10. (a) Prove that :

$$\int \cos^m x \cdot \sin^n x \, dx=\frac{\cos^{m-1}x \cdot \sin^{n+1}x}{m+n}+\frac{m-1}{m+1}\int \cos^{m-2}x \cdot \sin^n x \, dx.$$ 4

(b) Evaluate :

$$\int_0^1 \frac{1-4x+2x^2}{\sqrt{2x-x^2}}\, dx.$$ 3

(c) Prove that the area of an ellipse $b^2x^2 + y^2a^2 = a^2b^2$ is πab. 3

11. (d) If $\phi(n)=\int_0^{\pi/4} \tan^n x \, dx$ then prove that $\phi(n)+\phi(n-2)=\frac{1}{n-1}$ and hence find the value of $\phi(5)$. 4

(e) Evaluate $\int \frac{x^2+2x+3}{\sqrt{x^2+x+1}}\, dx$. 3

(f) Find the length of the arc of the curve $y=\log\left(\frac{e^x-1}{e^x+1}\right)$ from $x = 1$ to $x = 2$. 3

AS–1347

B.Sc. (Part—I) Semester—I Examination

MATHEMATICS

Paper—II

(Differential & Integral Calculus)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives (**1** mark each) :— 10

(i) If the function f(x) is differentiable at $x = x_o$, then it is :

(a) Not defined at $x = x_o$ (b) Continuous at $x = x_o$

(c) Not continuous at $x = x_o$ (d) None of these

(ii) The function f(x) has simple discontinuity if :

(a) $f(x^+)$, $f(x^-)$ do not exist

(b) $f(x)$, $f(x^+)$ $f(x^-)$ exist but not equal

(c) $f(x^+) = f(x^-) \neq f(x)$

(d) $f(x^+) \neq f(x^-)$

(iii) If $y = \sin(ax + b)$ then y_n is :

(a) $a^n \cos(ax + b + \frac{\pi}{2})$ (b) $a^n \sin(ax + b + \frac{\pi}{2})$

(c) $a^n \sin(ax + b + \frac{n\pi}{2})$ (d) $a^n \sin(ax + b - \frac{n\pi}{2})$

(iv) The graph of function $y = f(x)$, $\forall x \in [a, b]$ which satisfies all conditions of Rolle's theorem then geometrically, there exists at least one point c on the curve between $x = a$ and $x = b$ at which the tangent to the curve is :

(a) Parallel to y-axis (b) Parallel to x-axis

(c) Perpendicular to x-axis (d) Perpendicular to y-axis

(v) The expansion of the function e^x is :

(a) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+....$ (b) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+....$

(c) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....$ (d) $x-\frac{x^3}{3!}+\frac{x^5}{5!}+....$

(vi) The value of $\int \log x \, dx$ is :

(a) $\log x + k$ (b) $x \log x - x + k$

(c) $x \log x + x + k$ (d) $\log x - x + k$

(vii) The value of $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$ is :

(a) $e^{\frac{1}{2}}$ (b) $e^{-\frac{1}{2}}$

(c) 0 (d) 1

(viii) The degree of homogenous function, $f(x, y) = \sqrt{\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{12}$ (d) $\frac{1}{2}$

(ix) If $I_n = \int \sec^n dx$ then the reduction formula for I_n is :

(a) $I_n = -\frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$

(b) $I_n = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$

(c) $I_n = \frac{1}{n-1} \sec^{n-1} x \tan x - \frac{n-2}{n-1} I_{n-2}$

(d) $I_n = \frac{1}{n+1} \sec^{n+1} x \tan x - \frac{n-2}{n-1} I_{n-2}$

(x) Let f(x) be continuous and non-negative on [a, b]. Then the area A bounded by curve y = f(x), the x-axis and two ordinates x = a, x = b is :

(a) $A = \int_a^b x\,dx$ (b) $A = \int_a^b y\,dx$

(c) $\int_b^a y.dx$ (d) $\int_{-a}^{-b} f(x)\,dx$

UNIT—I

2. (a) Prove that if $\lim_{x \to x_0} f(x)$ exists, then it is unique. 4

(b) Using the $\in-\delta$ definition, show that $\lim_{x \to 2} x^2 = 4$. 3

(c) Show that $f(x) = \frac{1}{1-e^{\frac{1}{x}}}$ has simple discontinuity at x = 0. 3

3. (a) Prove that if f(x) is defined and continuous in [a, b], then it attains its bounds at least once in [a, b]. 4

(b) Using $\epsilon-\delta$ definition, prove that f(x) = sin x is continuous for all real values of x. 3

(c) Prove that $\lim_{x\to 2} f(x) = 7$, where f(x) = 2x + 3, $\forall$ x $\in$ [0, 5]. 3

UNIT—II

4. (a) Prove that : If f(x) is differentiable at $x = x_o$, then it is continuous at $x = x_o$. Is converse of this statement true ? Justify. 4

(b) Find the n^{th} differential coefficient of $\frac{1}{6x^2 - 5x + 1}$. 3

(c) If $y = x^n \log x$, then show that $y_{n+1} = \frac{n!}{x}$. 3

5. (a) State and prove Leibnitz's theorem. 4

(b) Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{\tan 5x}{\tan x}$. 3

(c) Prove that $\lim_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right] = -\frac{1}{2}$. 3

UNIT—III

6. (a) State and prove Lagrange's mean value theorem. 4

(b) Verify Cauchy mean value theorem for the functions :

$f(x) = e^x$ and $g(x) = e^{-x}$ in [a, b]. 3

(c) Expand $3x^3 + 4x^2 + 5x - 3$ about the point x = 1 by Taylor's theorem. 3

7. (a) State and prove Rolle's theorem. 4

(b) Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$. 3

(c) By using Lagrange's mean value theorem, show that $1 + x < e^x < 1 + xe^x, \forall\ x > 0$. 3

UNIT—IV

8. (a) Let F(u) be a homogenous function of degree n in x and y, where u is function of x and y. Then prove that :

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{nF(u)}{F'(u)} = G(u)$ and

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = G(u)[G'(u) - 1]$. 4

(b) If $u = \frac{x^2 + y^2}{x + y}$, prove that $(u_x - u_y)^2 = 4[1 - u_x - u_y]$. 3

(c) Verify Euler's theorem on homogeneous function for $u = \log\left[\frac{x+y}{x-y}\right]$. 3

9. (a) If $u = F(x - y, y - z, z - x)$, then prove that :

$u_x + u_y + u_z = 0$. 4

(b) If $u = \sin^{-1}\left\{\frac{x^2 + y^2}{x + y}\right\}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. 3

(c) Show that : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$, if $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$. 3

UNIT—V

10. (a) Prove that $\int \cot^n x \, dx = -\frac{1}{n-1}\cot^{n-1} x - I_{n-2}$. Hence evaluate $\int \cot^5 x \, dx$. 4

(b) Calculate the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 3

(c) Evaluate : $\int \frac{2x^2+3x+7}{\sqrt{x^2+x+1}} dx$. 3

11. (a) Prove that : $\int \sin^m x \cos^n x \, dx = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x \, dx$. 4

(b) Find the area between the curve $y = x^3 - 3x^2 + 2x$ and the x-axis. 3

(c) Find the length of the arc of the parabola $x^2 = 4ay$ from the vertex to an extremity of the latus rectum. 3

First Semester B. Sc. (Part - I) Examination

MATHEMATICS

Paper - II

(Differential and Integral Calculus)

P. Pages : 8

Time : Three Hours] [Max. Marks : 60

Note : (1) Question No. **One** is compuslsory attempt once.

(2) Attempt **One** question from ech units.

1. Choose the correct alternatives (1 mark each):—

(i) $\lim_{x \to 0} \frac{1}{x} \cos \frac{1}{x} =$ ------

(a) Limit exist.

(b) Limit does not exist.

(c) Equal to zero.

(d) None of these.

(ii) The function f is defined by f(x) = tan x is discontinuous at ————

(a) $x = \frac{\pi}{2}$ only

(b) $x = n\pi$. $\forall n \in \pi$

(c) $x = \frac{n\pi}{2}$, $\forall$ $n \in N$

(d) $x = (2n+1)\frac{\pi}{2}$, $n = 0, 1, 2,$ ----

(iii) The modulus function $f(x) = |x|$, $\forall x \in R$ is ———— at $x = 0$

(a) Continuous but not derivable.

(b) Derivable but not continous.

(c) Continuous and derivable.

(d) None of these.

(iv) $\lim_{x \to 0} \frac{\sin x}{x} =$ ------

(a) 0 (b) 1

(c) ∞ (d) None of these

(v) Let f be differentiable function on (a, b). Then which of the following statement is correct :

(a) $f'(x) \geq 0, \forall x \in (a,b) \Rightarrow f$ is monotone decreasing.

(b) $f'(x) = 0, \forall x \in (a,b) \Rightarrow f$ is not constant.

(c) $f'(x) \leq 0, \forall x \in (a,b) \Rightarrow f$ is monotone decreasing

(d) $f'(x) \leq 0 \ \forall x \in (a,b) \Rightarrow f$ is not decreasing.

(vi) The series $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \text{----} + \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + \text{------}$ is called-------

(a) Taylor's series.

(b) Maclaurin's series.

(c) Lagranges series.

(d) None of these.

(vii) If $u = \frac{x^4 - y^4}{x - y}$, $x \neq y$ then $xu_x + yu_y =$----

(a) 1.u　　(b) 4u

(c) 3u　　(d) None of these

(viii) Let $f(x, y) = 2x^3y^2 - 3xy^2 + x - 2y$ then f_{yy} = ________

(a) $4x^3 - 6x$　　(b) $12x^2 - 6y$

(c) $4y - 6$　　(d) None of these

(ix) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) πa (b) πab

(c) $\sqrt{\pi}\, ab$ (d) None of these

(x) $\int_0^{\pi/8} \cos^3 4x \, dx =$ ———

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{8}$ 10

UNIT I

2. (a) Let F(x) and g(x) be defined at all points of an interval [a, b] except possibly at $x_0 \in [a, b]$. If $\lim_{x \to x_0} F(x) = l$, $\lim_{x \to x_0} g(x) = m$, then prove that

$$\lim_{x \to x_0} [f(x) + g(x)] = l + m$$

(b) Show that $f(x) = \begin{cases} \frac{e^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

has a simple discontinuity at $x = 0$. 3

(c) Prove that $F(x) = x^2$ is continuous at $x = 3$. 3

3. (P) Show that the function $f(x)=\begin{cases}(1+2x)^{1/x} & x\neq 0\\ e^2 & , x=0\end{cases}$

is continuous at x = 0 3

(q) Prove that $\lim_{x\to a} \sin x = \sin a$ by ϵ–δ definition. 4

(r) If a function f is continuous on the closed interval I = [a, b] and f(a) ≠ f(b), then f assumes every value between f(a) and f(b). 3

UNIT II

4. (a) Justify, by an example, that continuity of a function at a point necessarily not imply the derivability at that point. 4

(b) If $y = \frac{1}{x^2+a^2}$, then find y_n. 3

(c) Evaluate $\lim_{x\to 0} (\cos x)^{1/x^2}$ 3

5. (p) Find the right hand and left hand derivative of $f(x) = |x|$ at x = 0 3

(q) If y = a cos(log x) + b sin(log x), show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. 4

(r) Evaluate $\lim_{x \to \frac{\pi}{2}} \left(\frac{\tan 5x}{\tan \cdot x} \right)$ 3

UNIT III

6. (a) State and prove Rolle's mean value theorem. 4

   (b) Verify Cauchy mean value theorem for $f(x) = \cos x$ $g(x) = \sin x$ in $[0, \pi/2]$ 3

   (c) Show that :

   $$\log(x+h) = \log h + \frac{x}{h} - \frac{x^2}{2h^2} + \frac{x^3}{3h^3} - \text{----}$$

7. (p) Let f be differentiable on (a, b) then Prove that $f(x) \geq 0 \ \forall \ x \in (a, b) \Longrightarrow f$ is monotone increasing. 3

   (q) Expand $2x^3 + 7x^2 + x-1$ in power of $(x-2)$. 3

   (r) State and prove Lagrange's mean value theorem. 4

UNIT IV

8. (a) If $u = \frac{x^2+y^2}{x+y}$ Prove that

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$$ 3

(b) Let F(u) be a hamogeneous function of degree n in x and y, where u is a function of x, y. Then :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{nF(u)}{F'(u)}$$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = G(u)[G'(u)-1]$$

where $G(u) = \frac{nF(u)}{F'(u)}$ and suitable condition of differentiability. 4

(c) If $u = \log(x^3+y^3+z^3-3xyz)$, show that

$$u_x+u_y+u_z = \frac{3}{x+y+z}$$ 3

9. (p) If $Z = f(xy)$, show that $xz_x - yz_y = 0$

(q) Verify Euler's theorem on homogeneous functions for $3x^2yz + 5xy^2z + 4z^4$. 4

(r) If $u = \dfrac{e^{x+y+z}}{e^x+e^y+e^z}$ then show that

$u_x + u_y + u_z = 2u$ 3

UNIT V

10. (a) If $I_n = \int \sec^n x \, dx$, then prove that

$$I_n = \frac{1}{n-1} \sec^{n-2} x \cdot \tan x + \frac{n-2}{n-1} I_{n-2}$$ 4

(b) Find the area between the curve

$y = x^3-3x^2+2x$ and the x–axis. 3

(c) Prove that $\int_0^1 x^{3/2}(1-x)^{3/2} \, dx = \dfrac{3\pi}{128}$ 3

11. (p) If $\phi(n) = \int_0^{\pi/4} \tan^n x \, dx$, then show

that $\phi(n) + \phi(n-2) = \dfrac{1}{n-1}$ and find the value of $\phi(5)$ 4

(q) Show that in the catenary $y = C \cosh \frac{x}{c}$, the length of the arc from the vertex to any print is given by $S = C \sin h \frac{x}{a}$. 3

(r) Integrate $\int \dfrac{2x^2-x+18}{\sqrt{x^2-2x+17}} \, dx$ 3